Finite Element Modeling of Electromechanical Transducers

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Finite Element Modeling of Electromechanical Transducers

Reinhard Lerch

6.00 p.m. Begin

Introduction

- Motivation for Computer modeling
- Electromechanical transducers and their numerical analysis

Theoretical background

- Finite element method (FEM)
- Boundary element method (BEM)
- FEM-BEM
- Open domain problems
- Coupled field problems within transducers:
  - Piezoelectricity
  - Electrostatic-Mechanic
  - Magneto-Mechanic
  - Fluid-Solid

Finite Element Modeling of Electromechanical Transducers

Reinhard Lerch

General information

- From physical reality to FE-model
- Pre- and Postprocessing (CAE environment)
- Material parameters
- Computational power over last 20 years
- Fast computation using Multigrid Methods
- Available codes

7.00 p.m. break/short discussion

Finite Element Modeling of Electromechanical Transducers

Reinhard Lerch

7.10 p.m. Acoustics

- Solution of wave propagation problems
- A simple example: plane wave radiation
- Wave propagation in flowing media
- Sound barrier
- Ultrasonic flow meter
- Nonlinear acoustics

7.40 p.m. break/short discussion

Finite Element Modeling of Electromechanical Transducers

Manfred Kaltenbacher

7.45 p.m. Piezoelectric transducers

- Piezoelectric finite elements
- Some simple examples
  - Impedance calculations
  - Eigenfrequencies: zero-coupling modes
- Annular array
  - FEM-BEM modeling scheme
  - Radiated sound fields
- Ultrasonic phased array antenna
  - Cross-talk
  - Pressure pulse
  - Pulse-echo simulations

- Surface Acoustic Wave (SAW) transducers
- Nonlinear piezoelectric material modeling
- Piezoelectric stack actuator

Finite Element Modeling of Electromechanical Transducers

Manfred Kaltenbacher

8.35 p.m. Electrostatic transducers

- Electrostatic-mechanical coupling
- Moving body within an electric field
- Iterative solution algorithm
- Voltage driven bar
- Capacitive micromachined ultrasound transducers
- Mirror actuator

9.05 p.m. break/short discusssion

Finite Element Modeling of Electromechanical Transducers

Manfred Kaltenbacher

9.10 p.m. Magnetomechanical transducers

- Magnetic field computation
- Eddy current sensor
- Electromagnetic-mechanical coupling
- Moving body in a magnetic field
- Electromagnetic acoustic transducer (EMAT)
- Electrodynmamic loudspeaker
- Sound emission of loaded power transformer
- Electromagnetic valve

9.50 p.m. Final Discussion

10.00 p.m End

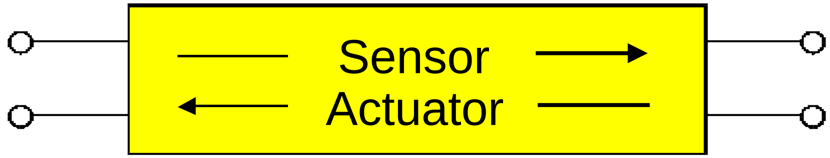
What is an

Electromechanical Transducer

and

how can it be modeled?

Electromechanical Transducers



Mechanics

- Distance
- Velocity
- Acceleration
- Force
- Pressure
- Torque
- Mass Flow
- Acoustic quantities

- Piezoelectric
- Piezoresistive
- Electrostatic
- Elektrodynamic
- Magnetomechanic
- Optical

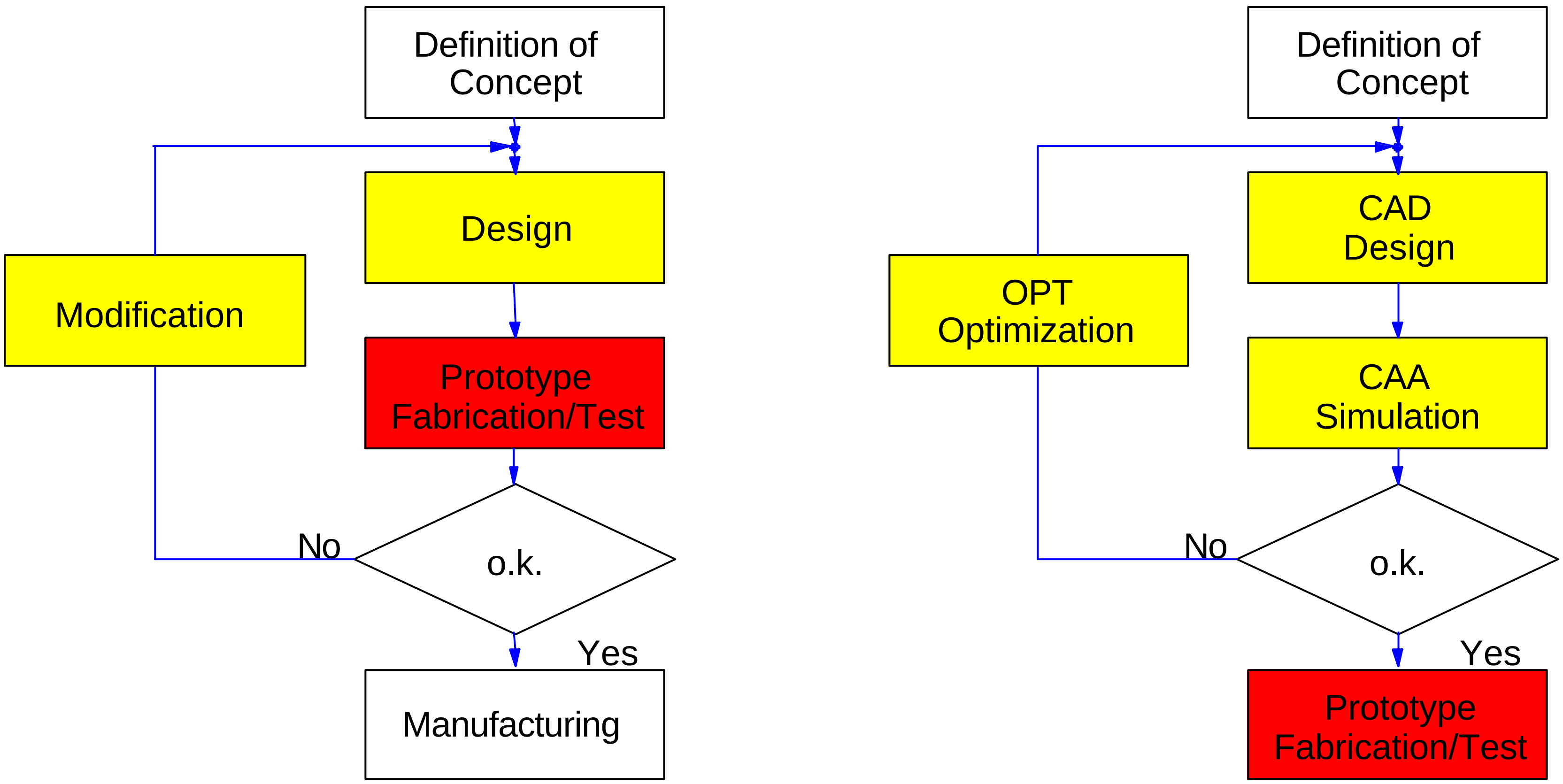
Electrics

- Voltage
- Current
- Charge
- Impedance

Features of Electromechanical Sensors and Actuators

- Interdisciplinary, since they are based on the interaction of different fields, e.g. magnetic and mechanical field
- High complexity, e.g. microsystems
- Variety of variants
- Short product life time cycles

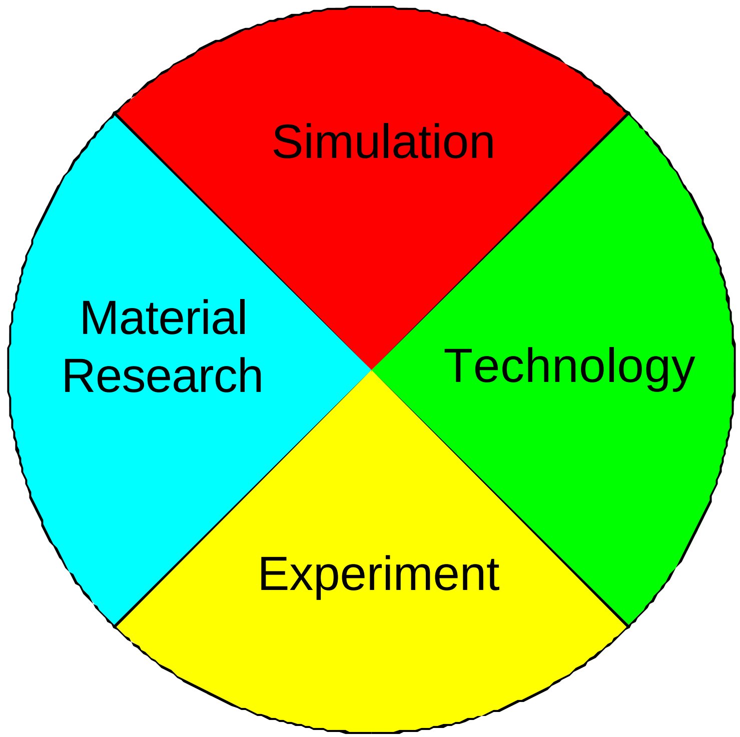
Development Methodologies



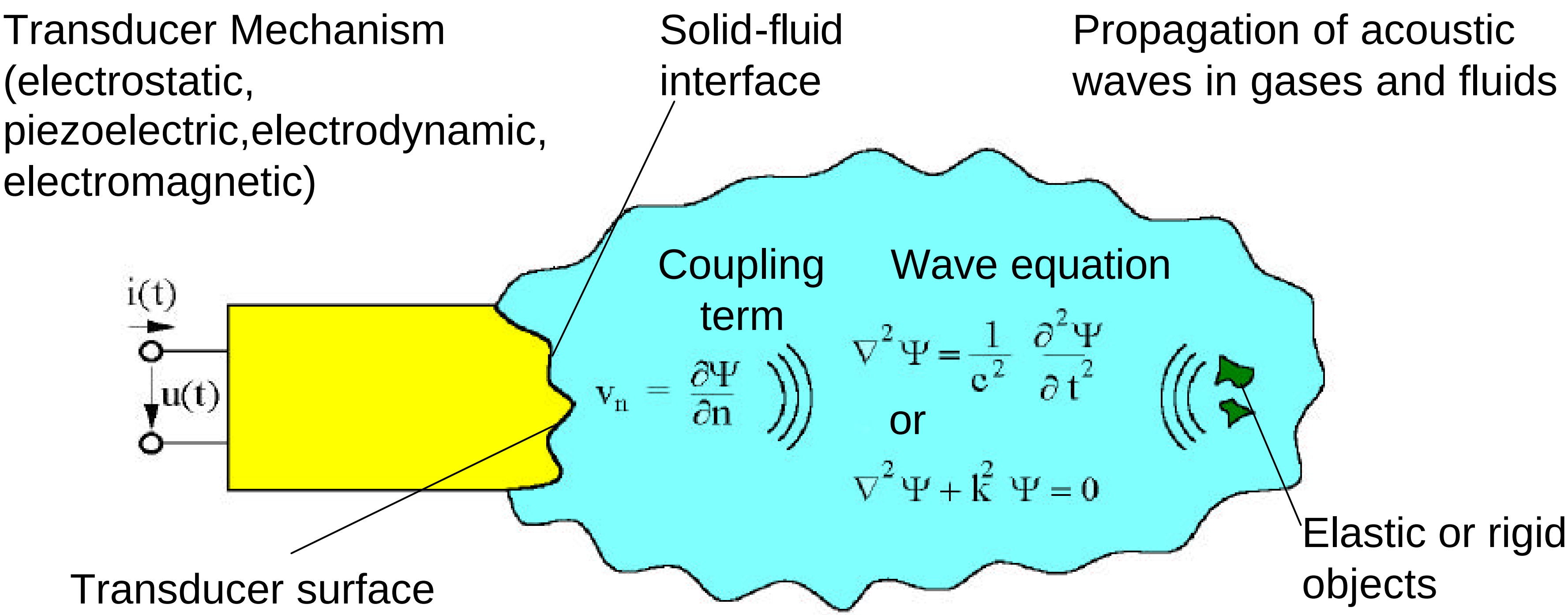
Benefits of Computer Simulations within Transducer Development

- Design with minimum hardware effort
  - shorter design cycle
  - reduced costs
- Simultaneous Engineering
- Isolation of design parameters
- Clean design environment without external disturbances
- Learning by simulation for a better basic understanding
- Optimization with higher quality

Concept of Transducer Development



Electromechanical Transducer (Sensor or Actuator)



Basic Equations for Electromechanical Transducers

- Mechanics
  - Hooke’s Law
  - Newton’s Law
- Electromagnetics
  - Maxwell’s equations without displacement currents (eddy current case)
- Thermodynamics

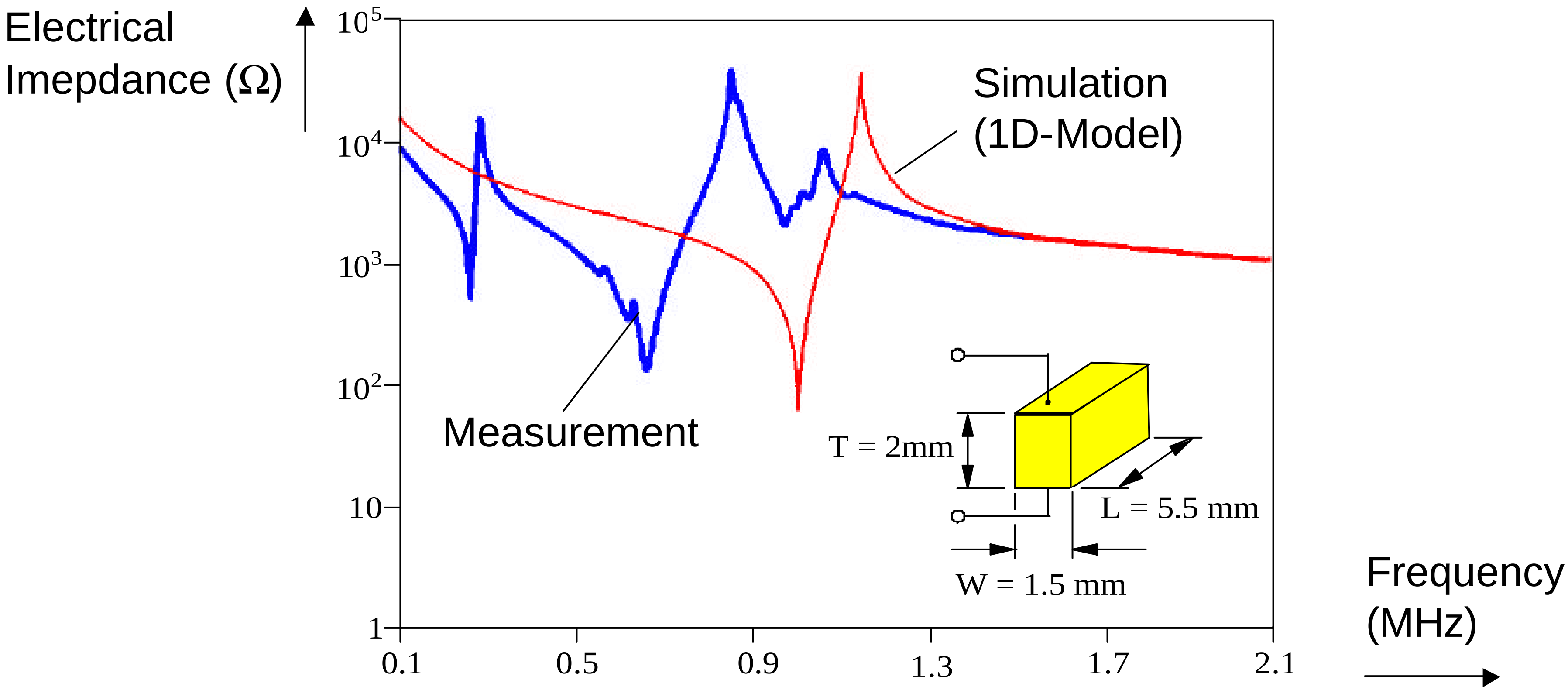
  Equations describing
  - Thermal expansion
  - Thermal conductivity
  - Convection

Coupled Field Problems within Electromechanical Transducers

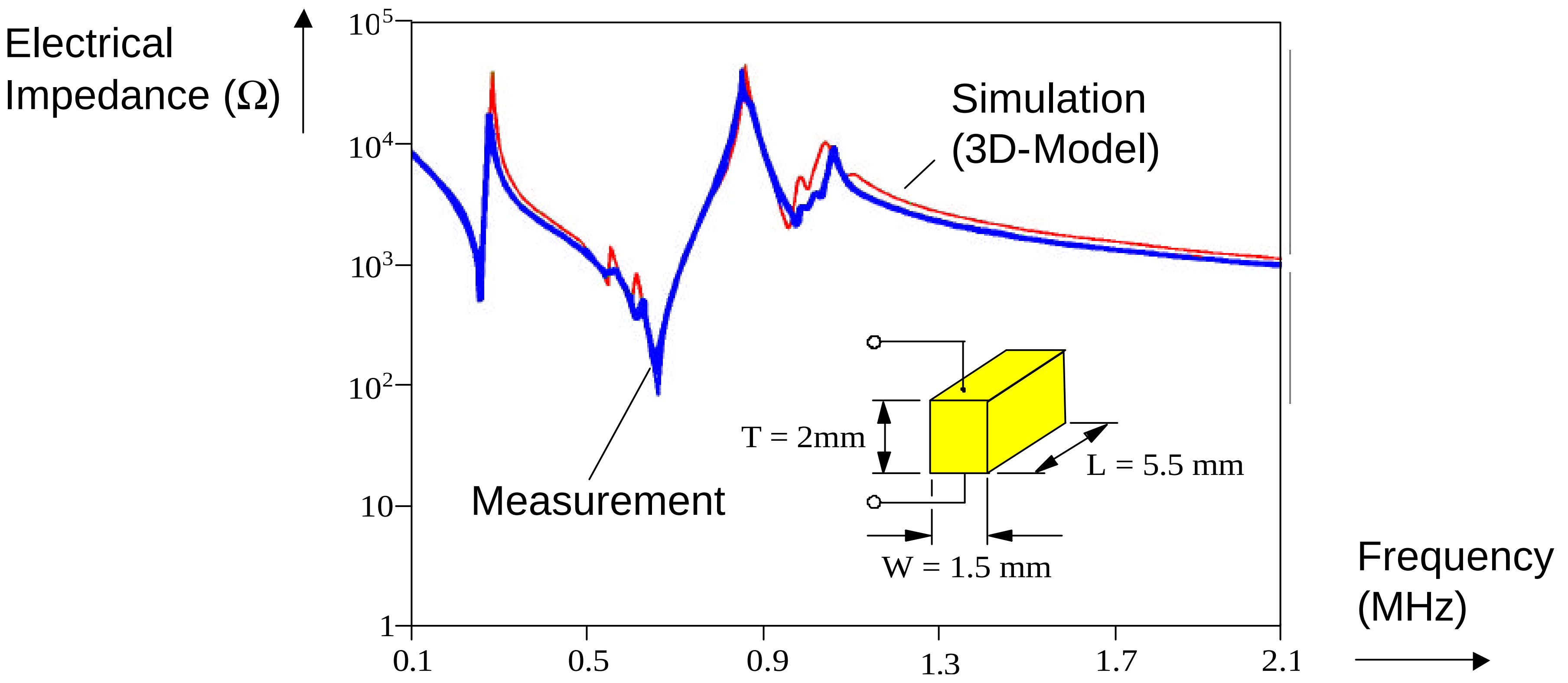
- Fluid-Solid Coupling
- Electrostatics-Mechanics (Coulomb force)
- Piezoelectricity (direct and inverse piezoeffect)
- Piezoresistivity and Piezojunction effect
- Electromagnetics-Mechanics (Lorentz force, magnetic force, magnetostriction, electromagnetic induction)
- Thermoelectrics (pyroelectricity, heat generation due to conductivity loss)
- Temperature creep

Which type of Modeling do we need in Transducer Design ?

Electrical Impedance of a Piezoceramic Array Transducer



Electrical Impedance of a Piezoceramic Array Transducer



We need a
3D Modeling Scheme for
Coupled Field Problems

- General applicability and flexibility
- Interfaces to standard CAD systems

Finite Element Method (FEM), or
Boundary Element Method (BEM), or
FE/BE Method

How do Finite Elements work ?

Finite Element Analysis of Electrostatic Problems

- Electrical fields (f and $\vec{E}$) within (anisotropic) dielectrica
- Arbitrary geometry of
  - dielectrica
  - electrodes
- Arbitrary charge distributions
- Computation of capacitances between electrodes

Finite Element Method (FEM) for Potential Equation (I)

- Potential Equation

$$\nabla \cdot \epsilon \nabla \phi = q \tag{1}$$

ϕ : electric potential

ϵ : scalar electric permittivity, assumed to be constant

q : electric volume charge

The equation above has to hold in a closed bounded body Ω with smooth surface Γ, where the homogenous boundary condition $\phi = 0$ holds.

FEM for Potential Equation (II)

- Step 1: Test Functions

  Multiplying equation (1) by ω and integrating over Ω gives

$$\int_\Omega \omega \nabla \cdot \epsilon \nabla \phi \, d\Omega = \int_\Omega \omega q \, d\Omega \quad (2)$$

ω an arbitrary, smooth function on Ω, which vanishes on Γ (test function)

- Step 2: Green's Identity

  For ω and ϕ Green's identity holds.

$$\int_\Omega \nabla \omega \cdot \epsilon \nabla \phi \, d\Omega = \int_\Gamma \omega \epsilon \frac{\partial \phi}{\partial n} \, d\Gamma - \int_\Omega \omega \nabla \cdot \epsilon \nabla \phi \, d\Omega \quad (3)$$

n outer unit normal on Γ and $\frac{\partial \phi}{\partial n} = \nabla \phi \cdot n$ the normal derivative of ϕ

FEM for Potential Equation (III)

- Step 3: Weak Formulation
  Using Green's identity in equation (2) we obtain:

$$\int_{\Omega} \nabla\omega \cdot \epsilon\nabla\phi \, d\Omega = \int_{\Gamma} \omega\epsilon\frac{\partial\phi}{\partial n} \, d\Gamma - \int_{\Omega} \omega q \, d\Omega \qquad (4)$$

  and since ω vanishes on Γ

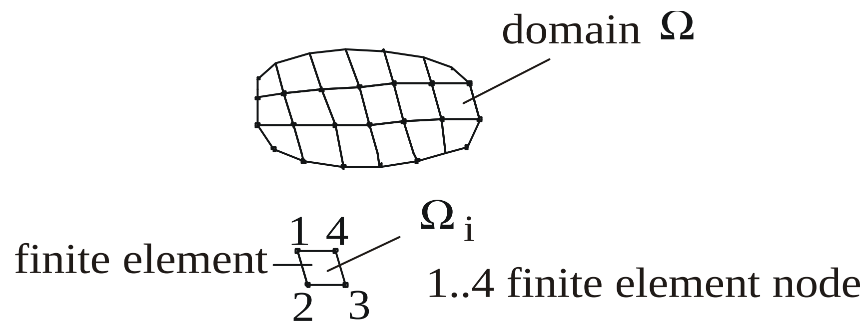
$$\int_{\Omega} \nabla\omega \cdot \epsilon\nabla\phi \, d\Omega = -\int_{\Omega} \omega q \, d\Omega \qquad (5)$$

  Equation (5) is called the weak form of equation (1).

FEM for Potential Equation (IV)

- Discretization

  Divide Ω into small bodies Ω_i



$$\Omega = \sum_{i=1,n} \Omega_i$$

  the finite elements.
  Each Ω_i is of simple geometric shape, such as triangles or quadrilaterals in 2D and tetrahedra or hexadra in 3D.
  The vertices of Ω_i are the nodes P_j^i

  For *P* Î Ω_i:

$$P = \sum_j N_j(P) P_j^i$$

$$\phi(P) = \sum_j N_j(P) \phi(P_j^i)$$

FEM for Potential Equation (V)

- System Equation

$$\mathbf{K}\{\phi\} = \{Q\}$$

K electrical stiffness matrix

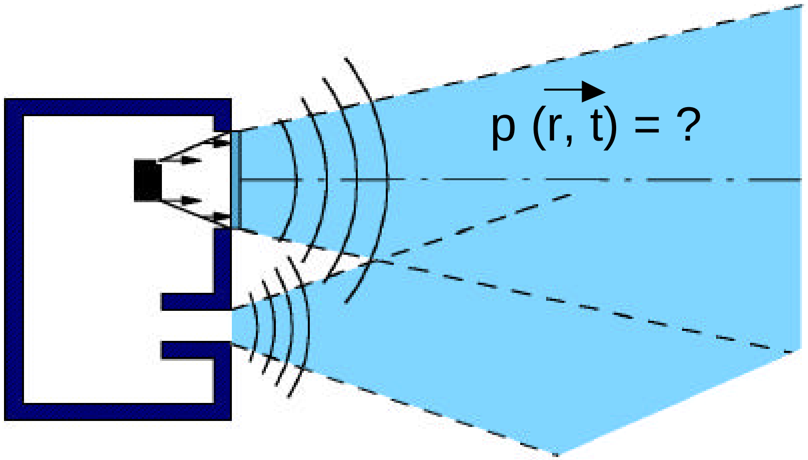
(sparse and symmetric matrix)

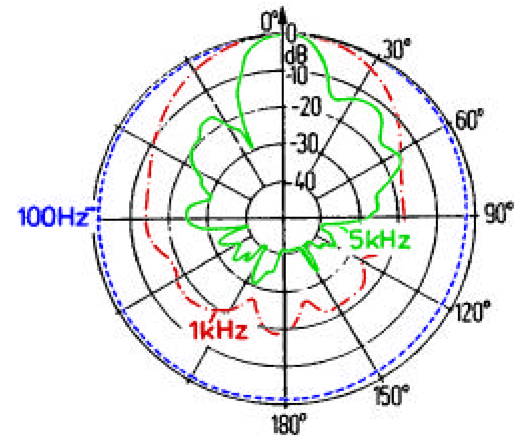
$\{\phi\}$ vector of nodal potentials

$\{Q\}$ vector of applied external charge loads

Finite/Boundary Element Analysis of Acoustic Problems

Radiation of Vibrating Objects





- Acoustic pressure fields
- Acoustic velocity fields
- Radiation patterns
- Power flow characteristics
- Interaction with solids, e. g. diffraction effects due to rigid/elastic objects
- Sound in enclosures
- Sound in media with flow

Computational Acoustics

Basic Methods:

- Finite Elements (FE)
- Boundary Elements (BE)
- Huygens-Method
- Hybrid Methods
  - FE/BE
  - FE/Huygens

Computational Acoustics – Finite Element Approach

Application Examples:

- Ultrasound transducers
- Microphones and loudspeakers
- Ultrasound flowmeters
- Car engines
- Sound emission by transformers
- Sound protection walls

Finite Element Methods in Acoustics (I)

- Wave equation

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

ψ acoustic potential with

$$\vec{v} = -\nabla\psi, \qquad p = \rho\frac{\partial\psi}{\partial t}$$

- Weak formulation

$$\int_\Omega \nabla\omega \cdot \nabla\psi \, d\Omega - \int_\Omega \frac{1}{c^2}\omega \frac{\partial^2\psi}{\partial t^2} d\Omega = 0$$

Ω closed body with smooth surface Γ, ω differentiable and vanishing on Γ

Finite Element Methods in Acoustics (II)

- Discretization

Divide Ω into small bodies Ω_i

$$\Omega = \sum_{i=1,n} \Omega_i$$

the finite elements.
Each Ω_i is of simple geometric shape, such as tetraeder or hexaeder.
The vertices of Ω_i are the nodes P_j^i

For P Î Ω_i:

$$P = \sum_j N_j(P) P_j^i$$

$$\psi(P) = \sum_j N_j(P) \psi(P_j^i)$$

Finite Element Methods in Acoustics (III)

- System of Ordinary Differential Equations

$$\mathbf{M}\{\ddot{\Psi}\} + \mathbf{K}\{\Psi\} = \{F\}$$

M mass-matrix

K stiffness matrix

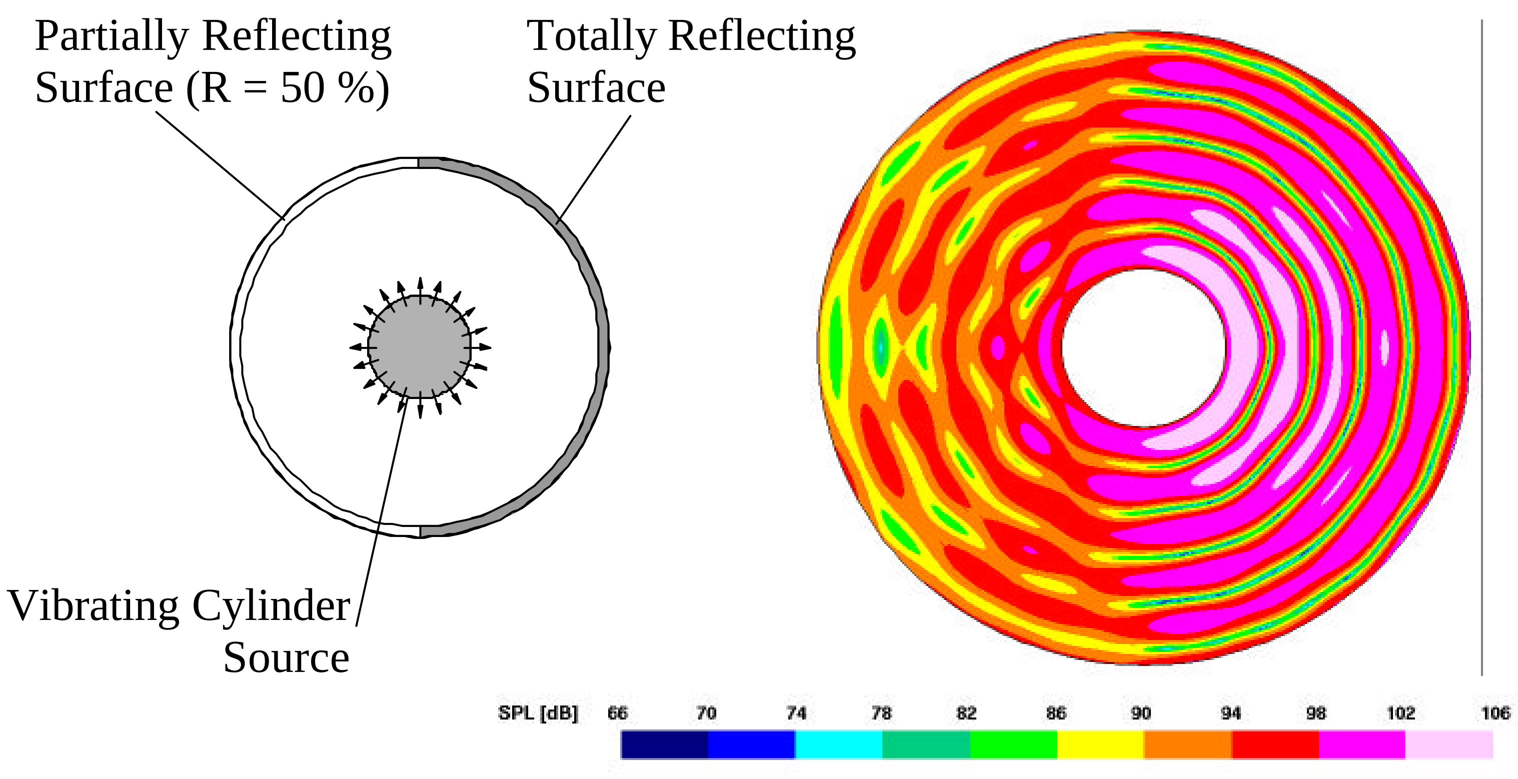
$\{\psi\}$ vector of nodal potentials

$\{F\}$ vector of applied external loads

Computational Acoustics – Finite Element Approach

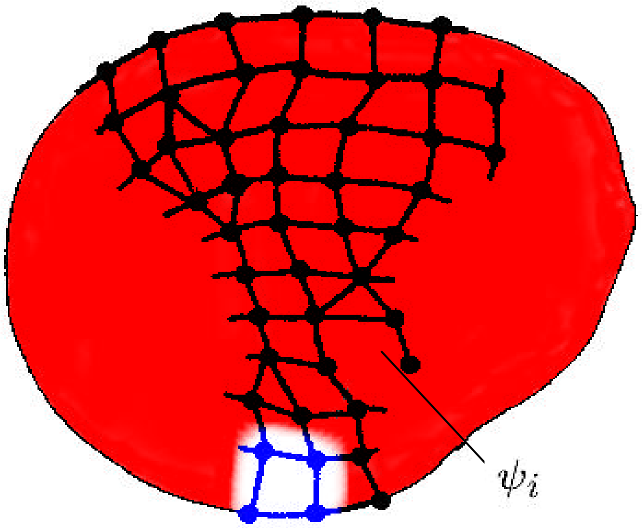
- 2 D and 3 D modeling
- Harmonic and transient analysis
- Infinite elements (radiation elements)
- Elements for partially absorbing surfaces
- Fluid-solid coupling
- Sound in media with flow

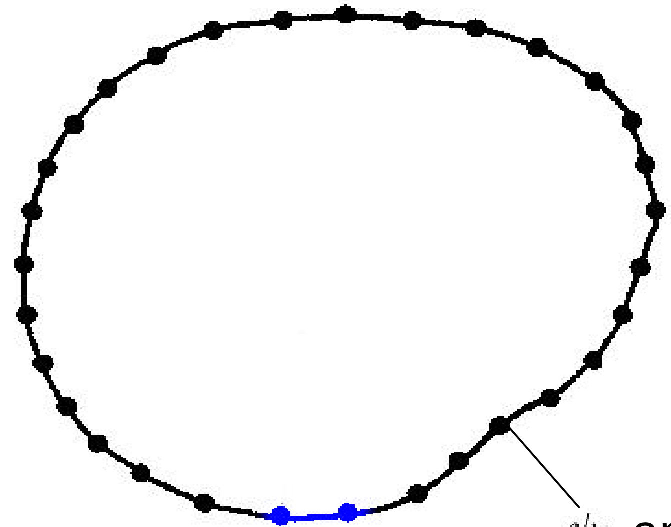
Surface with Partial Sound Absorption



Finite Elements vs. Boundary Elements (I)

Finite Element Method (FEM) **Boundary Element Method (BEM)**





ψ_i : scalar acoustic potential at node i

$\frac{\partial \psi_i}{\partial n}$: normal derivative of ψ_i (= normal velocity)

Direct BEM in Acoustics (I)

- Helmholtz's differential equation

$$\nabla^2 p + k^2 p = 0, \qquad k = w/c = 2\pi/\lambda,$$ wavenumber

p .. acoustic pressure

- Green's function

$$G(P,Q) = \frac{e^{-jkr}}{4\pi r}, \qquad r = dist(P,Q)$$

For *G* we have

$$\nabla_Q^2 G(P,Q) + k^2 G(P,Q) = \delta(P)\,,$$ δ (*P*) Dirac delta function

Direct BEM in Acoustics (II)

- Boundary integral formulation

$$\frac{1}{2}p(P) = \int_\Gamma p(Q)\frac{\partial}{\partial n}G(P,Q) + G(P,Q)\frac{\partial}{\partial n}p(Q)\, d\Gamma$$

Ω closed body with smooth surface Γ

- Discretization:divide Γ into small parts Γ_i,

$$\Gamma = \sum_{i=1,n} \Gamma_i$$

the boundary elements, with vertices P_j^i
For $P \in \Gamma_i$

$$p(P) = \sum_j N_j(P) P_j^i$$

$$\frac{\partial}{\partial n}p(P) = \sum_j N_j(P) \frac{\partial}{\partial n} P_j^i$$

Direct BEM in Acoustics (III)

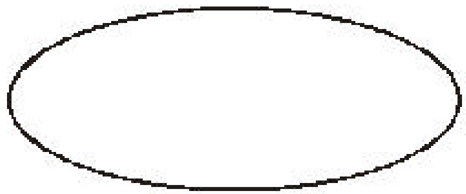
- Collocation
  Take *P* successively each node P_j :

$$\left[\mathbf{H} + \frac{\mathbf{1}}{\mathbf{2}}\right]\{p\} = [\mathbf{G}]\left\{\frac{\partial}{\partial n}p\right\}$$

Must be supplied with n boundary conditions.

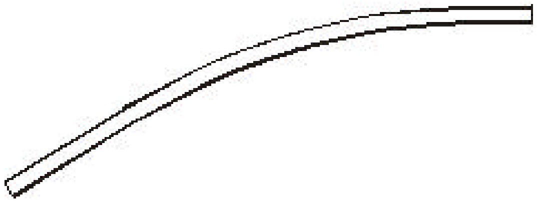
Indirect BEM is necessary for Modeling of Thin Structures

Direct BEM



- Unsymmetric system matrix
- Single surface integration

Indirect BEM



- Symmetric system matrix
- Double surface integration

Indirect BEM in Acoustics (I)

- Thin Structure Problem
  $\Gamma = \Gamma^+ \cup \Gamma^-$, boundary integral equation for $P \notin \Gamma$

$$p(P) = \int_{\Gamma^+} p(Q^+)\frac{\partial}{\partial n}G(P,Q^+) - G(P,Q^+)\frac{\partial}{\partial n}p(Q^+)\,d\Gamma + \int_{\Gamma^-} p(Q^-)\frac{\partial}{\partial n}G(P,Q^-) - G(P,Q^-)\frac{\partial}{\partial n}p(Q^-)\,d\Gamma$$

- Boundary Layer Potentials
  Now, $\Gamma = \Gamma^+ = \Gamma^-$. So

$$p(P) = \int_{\Gamma} \mu(Q)\frac{\partial}{\partial n}G(P,Q) - G(P,Q)\sigma(Q)\,d\Gamma$$

double layer potential $\quad \mu(Q) = p(Q^+) - p(Q^-)$

single layer potential $\quad \sigma(Q) = \frac{\partial}{\partial n}p(Q^+) - \frac{\partial}{\partial n}p(Q^-)$

Indirect BEM in Acoustics (II)

- For points on the surface

$$p(P^{\pm}) = \pm\mu(P) + \int_{\Gamma} \mu(Q)\frac{\partial G(P,Q)}{\partial n_Q} - G(P,Q)\sigma(Q)\, d\Gamma$$

$$\frac{\partial}{\partial n}p(P^{\pm}) = \pm\sigma(P) + \int_{\Gamma} \mu(Q)\frac{\partial^2 G(P,Q)}{\partial n_P \partial n_Q} - \frac{\partial G(P,Q)}{\partial n_P}\sigma(Q)\, d\Gamma$$

Indirect BEM in Acoustics (III)

- Vibrating Structures $\frac{\partial p}{\partial n} = -j\omega\rho v_n, \qquad \sigma(Q) = 0$

gives $$\int_\Gamma \mu(Q) \frac{\partial^2 G(P,Q)}{\partial n_P \partial n_Q} \, d\Gamma = -j\omega\rho v_n(P)$$

Hypersingular integral equation

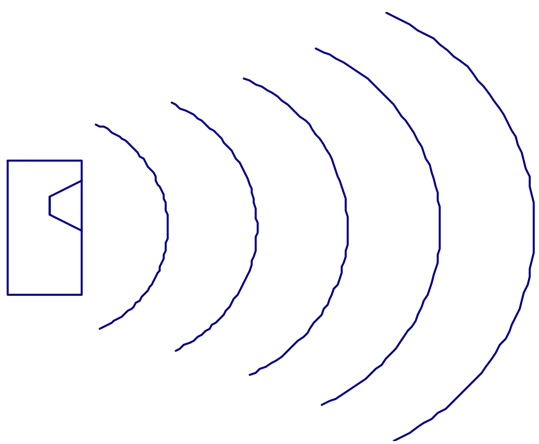
- Variational Formulation and Regularization

$$\int_\Gamma \int_\Gamma G(P,Q) \left[k^2 \mu(P)\mu(Q) \vec{n}_p \cdot \vec{n}_Q \; - \right.$$
$$\left. (\nabla \times \mu(P)) \cdot (\nabla \times \mu(Q)) \right] d\Gamma \, d\Gamma \; = \; \int_\Gamma -j\omega\rho v_n \, d\Gamma$$

Open Domain Problems (I)

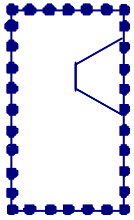
Problem

- Radiation into an open (unbounded) domain

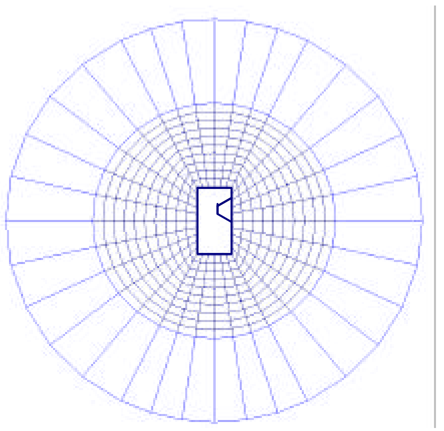


Solutions

- Boundary Elements



- Infinite Elements



Open Domain Problems (II)
– Infinite Elements –

Purpose: Absorption of wave energy travelling towards the open boundary (infinity)

Standard Implementation: Double Asymptotic Approximation

- Low frequencies: Mass loading dominates
- High frequencies: Sommerfeldt's radiation dominates

$$\lim_{r\to\infty} r\left(\frac{\partial\psi}{\partial n} + jk\psi\right) = 0$$

Open Domain Problems (III)
Infinite Elements vs. Boundary Elements

Boundary Elements: Ideal absorption
Continuous wave only (frequency domain)
Non-symmetric and fully populated system matrix
FEM/BEM solutions often necessary

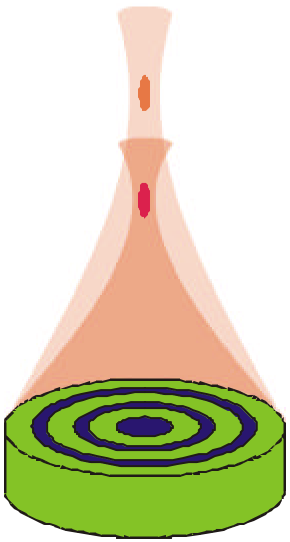
Infinite Elements: Absorption only approximately, i.e. partial reflections
depending on problem and modeling effort
Continuous wave as well as transient cases
(frequency and time domain)
Symmetric system matrix (standard FEM)
Pure FEM technique

Finite Elements vs. Boundary Elements (I)

FEM	BEM
Discretization of the whole region	Discretization of the region boundary only
Unbounded regions require special treatment (e.g. infinite elements)	Bounded and unbounded regions alike
Static, transient, harmonic, and eigenfrequency analysis	Transient analysis very inefficient
Result in a well-behaved system of ordinary differential equations	Leads to weak-, strong- or hypersingular integral equations
Simple numerical integration	Singular integrals
Resulting matrices are sparse and, in general, symmetric	Matrices are fully populated and unsymmetric (collocation) or symmetric (galerkin)

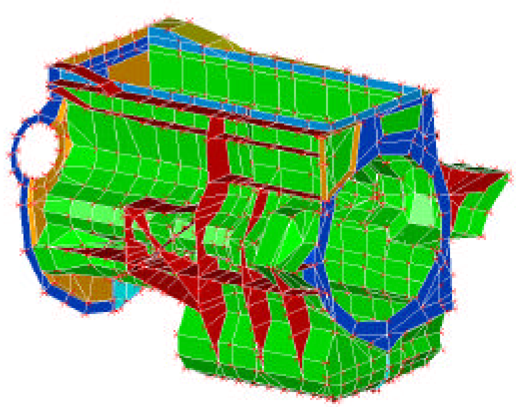
Finite Elements vs. Boundary Elements (II) – Ultrasonic Ring Antenna –

	FEM	BEM
Number of elements	10215	16
CPU-Time (min.)	80	3
Memory (Mbyte)	60	0,5
Accuracy (%)	2,5	1,5



Finite Elements vs. Boundary Elements (III)
– Sound Emission from a Diesel Engine –

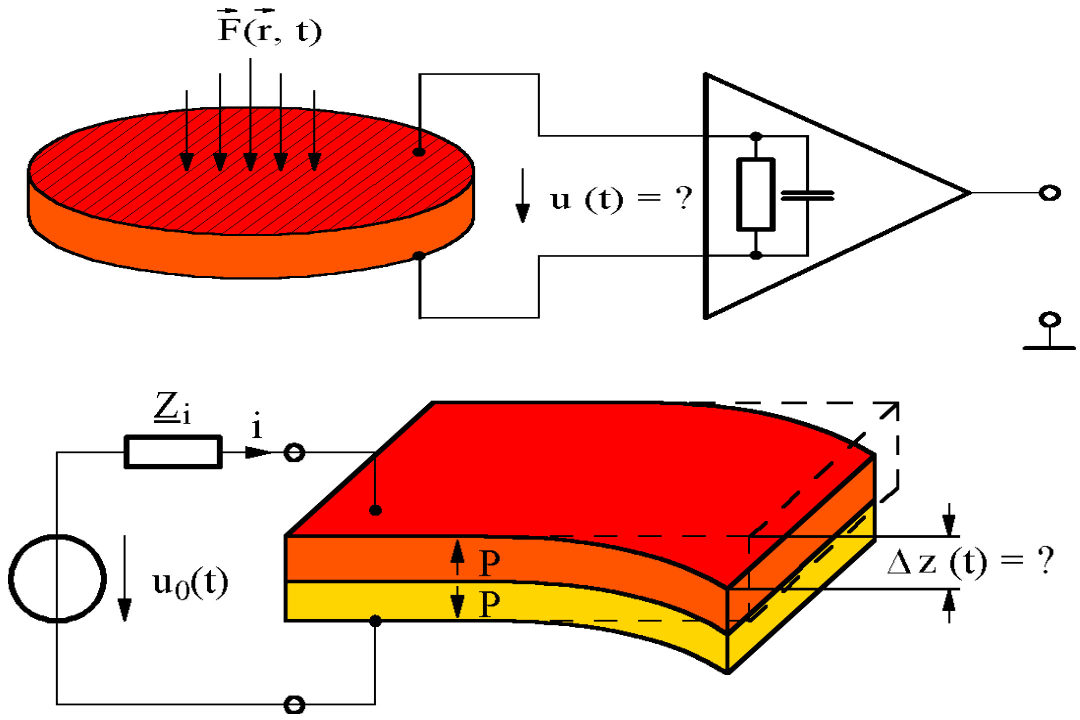
	BEM	FEM
2D-Elements on Surface	2600	2500
3D-Elements	–	220000
Main Memory (MB)	64	500
CPU Time for single Frequency	5h	–
CPU time for 1 ms	–	170 min
Overall solution time	600 h	28 h



Overall Solution Time:
128 frequencies (BEM)
10 ms (FEM)

Finite Element Analysis of Piezoelectric Sensors and Actuators

- Mechanical deformations and stresses
- Electrical voltages, charges, currents and impedances
- Electrical fields
- Distributions of electrical and mechanical energy
- Nonlinear behaviors



FE Modeling of Piezoelectric Transducers (I)

- Mechanical field

$$\nabla \vec{T} + \vec{f}_{\mathrm{V}} = \rho \frac{\partial^2 \vec{u}}{\partial t^2}$$

$$\vec{T} = [c]\vec{S}$$

$$\vec{S} = \mathbf{B}\vec{u}$$

$\vec{T}$	mechanical stress
$\vec{S}$	mechanical strain
$\vec{f}_{\mathrm{V}}$	volume force
$\mathbf{B}$	differential operator

- Coupling equations

$$\begin{aligned} \vec{T} &= [c]^E \ \vec{S} - [e]_t \ \vec{E} \\ \vec{D} &= [e] \ \vec{S} + [\varepsilon]^S \ \vec{E} \end{aligned}$$

$[c]^E$	mechanical material tensor
$[e]$	piezoelectric coupling tensor
$[\varepsilon]^S$	dielectric material tensor

- Electric field

$$\nabla \cdot [\varepsilon] \nabla \phi = 0$$

$\vec{D}$	electric displacement
$\vec{E}$	electric field intensity

FE Modeling of Piezoelectric Transducers (II)

- FE-formulation

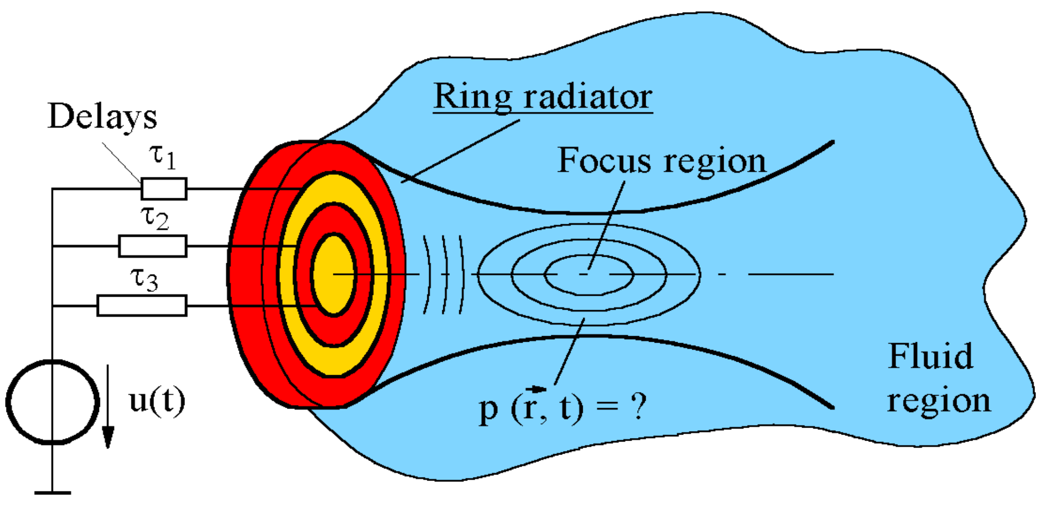
$$\begin{pmatrix} \mathbf{M}_{uu} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \{\ddot{u}\} \\ \{\ddot{\Phi}\} \end{pmatrix} + \begin{pmatrix} \mathbf{C}_{uu} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \{\dot{u}\} \\ \{\dot{\Phi}\} \end{pmatrix} + \begin{pmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} \\ \mathbf{K}_{u\phi}^t & -\mathbf{K}_{\phi\phi} \end{pmatrix} \begin{pmatrix} \{u\} \\ \{\Phi\} \end{pmatrix} = \begin{pmatrix} \{F\} \\ \{Q\} \end{pmatrix}$$

$\mathbf{K}_{uu}$	mechanical stiffness matrix
$\mathbf{C}_{uu}$	mechanical damping matrix
$\mathbf{M}_{uu}$	mechanical mass matrix
$\mathbf{K}_{\phi\phi}$	dielectric stiffness matrix
$\mathbf{K}_{u\phi}$	piezoelectric coupling matrix

$\{F\}$	external mechanical forces
$\{Q\}$	electric charges
$\{u\}$	nodal vector of displacement
$\{\Phi\}$	nodal vector of scalar electric potential

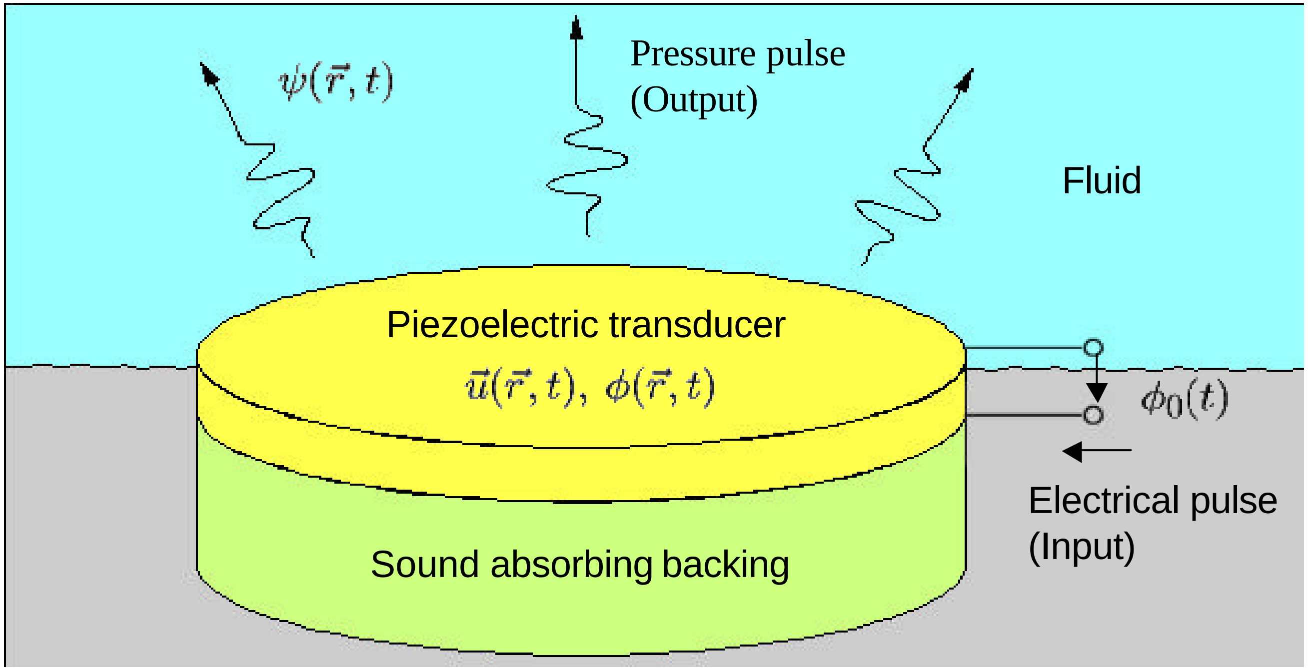
Finite Element Analysis of Piezoelectric Transducers with Fluid Load

Fluid loaded Piezoelectric Ring Antenna



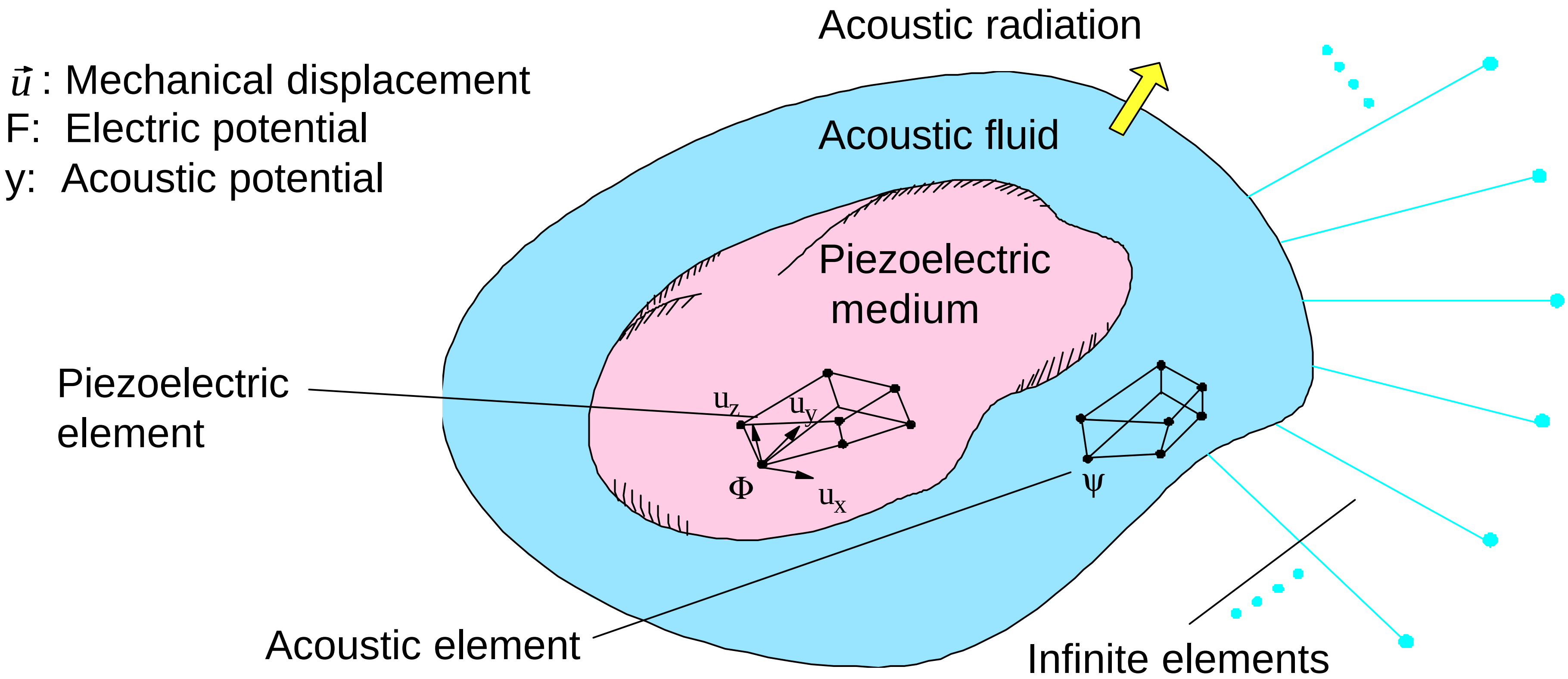
- Fluid loaded piezoelectric transducers for transmit, receive and, pulse-echo
- Fluid-solid interaction
- Pressure fields
- Diffraction effects
- Interaction of sound with elastic or rigid objects

Fluid-Solid Interaction in Ultrasonic Transducers



$\psi(\vec{r},t)$: scalar acoustic potential
$\vec{u}(\vec{r},t)$: mechanical displacement in solid
$\phi(\vec{r},t)$: electrical potential in piezoelectric solid

Finite Element Analysis of Interaction between Piezoelectric Solids and Acoustic Fluids



Finite Element Equations for Piezoelectric Media Immersed in an Acoustic Fluid

$$\begin{pmatrix} \mathbf{M}_{uu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\mathbf{M}_{\psi\psi} \end{pmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\Phi} \\ \ddot{\Psi} \end{Bmatrix} + \begin{pmatrix} \mathbf{C}_{uu} & 0 & \mathbf{C}_{u\psi} \\ 0 & 0 & 0 \\ \mathbf{C}^t_{u\psi} & 0 & -\mathbf{C}_i - \mathbf{C}_\psi \end{pmatrix} \begin{Bmatrix} \dot{u} \\ \dot{\Phi} \\ \dot{\Psi} \end{Bmatrix}$$

$$+ \begin{pmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} & 0 \\ \mathbf{K}^t_{u\phi} & -\mathbf{K}_{\phi\phi} & 0 \\ 0 & 0 & -\mathbf{K}_i - \mathbf{K}_\psi \end{pmatrix} \begin{Bmatrix} u \\ \Phi \\ \Psi \end{Bmatrix} = \begin{Bmatrix} F \\ Q \\ 0 \end{Bmatrix}$$

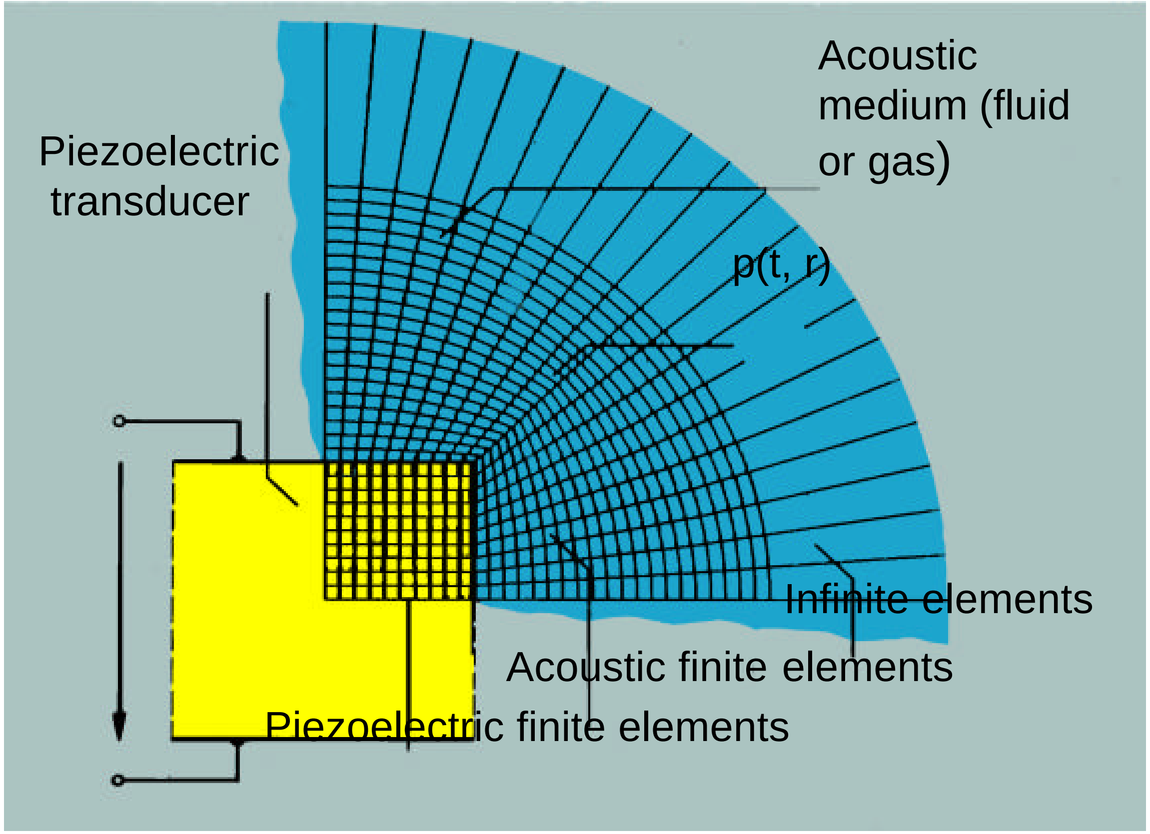
Nodal point vectors:

u : Mechanical displacement in piezoelectric solid

Φ: Electrical potential in piezoelectric solid

Ψ: Acoustic potential in fluid medium

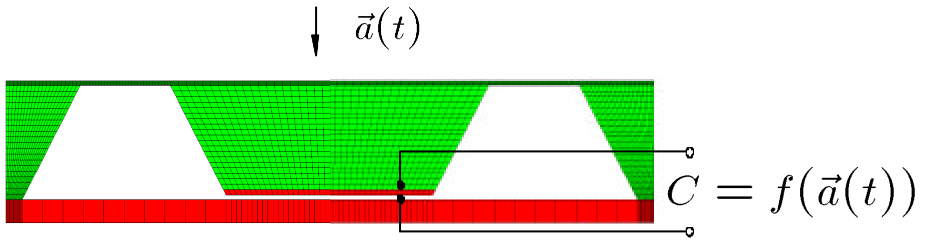
Finite Element Modeling of a Piezoelectric Transducer Immersed in an Acoustic Fluid



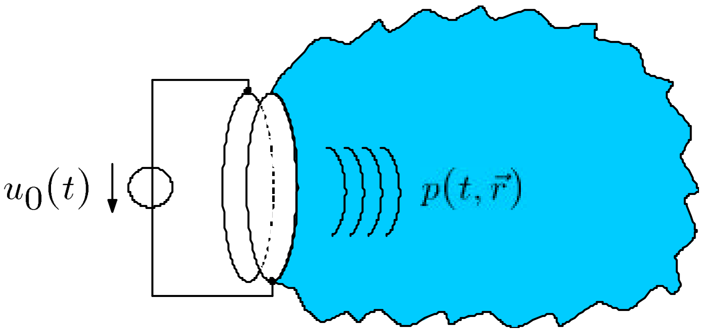
Finite/Boundary Element Analysis of Electrostatic Sensors and Actuators

- Full (nonlinear) coupling of mechanic and electric fields
- Calculation of:
  - Coulomb forces via electrostatic force tensor
  - Deformations and stresses
  - Electrical fields, charges and impedances
- Fluid-Solid-Coupling, e.g. for ultrasound transducers

Capacitive Acceleration Sensor



Ultrasonic Transmitter



FE/BE Modeling of Electrostatic Transducers

- Mechanical field is modeled by FE

$$\mathbf{M}\{\ddot{u}\} + \mathbf{C}\{\dot{u}\} + \mathbf{K}\{u\} - \{F_{\mathrm{mech}} + F_{\mathrm{el}}(\phi, \phi_{\mathrm{n}})\} = 0$$

- Electric field is modeled by BE

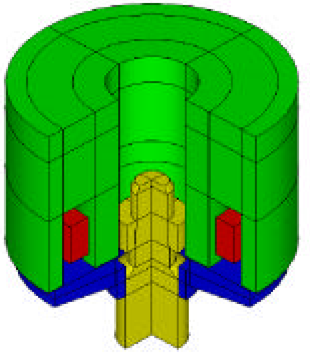
$$\mathbf{H}_{\phi}(u)\{\Phi\} - \mathbf{G}_{\phi}(u)\{\Phi_{\mathrm{n}}\} = \{Q\}$$

- Coupling between *electrostatic BE-equation and mechanical FE-equation via* Predictor/Multicorrector algorithm *within time step integration*

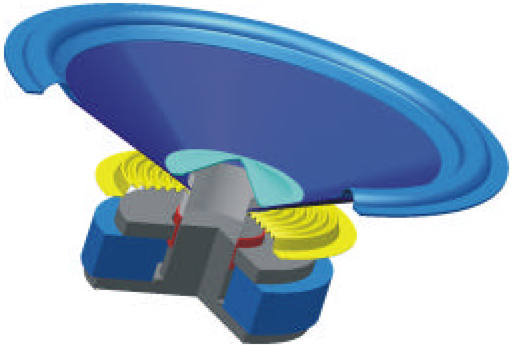
Finite/Boundary Element Analysis of Magnetomechanic Sensors and Actuators

- Full (nonlinear) coupling of mechanic and magnetic fields
- Calculation of
  - Lorentz forces
  - Voltages and fields induced by movement (e.m.f. terms)
- Nonlinear magnetization curves

Magnetic Valve



Electrodynamic loudspeaker



Considered Fields and their Couplings

i(t)

u(t)

Magnetic field

$$\nabla \times \left(\frac{1}{\mu}\nabla \times \vec{A}\right) = \vec{J}_e - \gamma\frac{\partial \vec{A}}{\partial t} - \gamma\nabla V + \gamma\vec{v} \times (\nabla \times \vec{A})$$

volume force $\vec{f}_V = \vec{J} \times (\nabla \times \vec{A})$ and motional emf-term $\gamma\,\frac{\partial \vec{d}}{\partial t} \times (\nabla \times \vec{A})$

Mechanical field

$$\frac{E}{2(1+\nu)}\left((\nabla \cdot \nabla)\vec{d} + \frac{1}{1-2\nu}\nabla(\nabla \cdot \vec{d})\right) + \vec{f}_V = \rho\frac{\partial^2 \vec{d}}{\partial t^2}$$

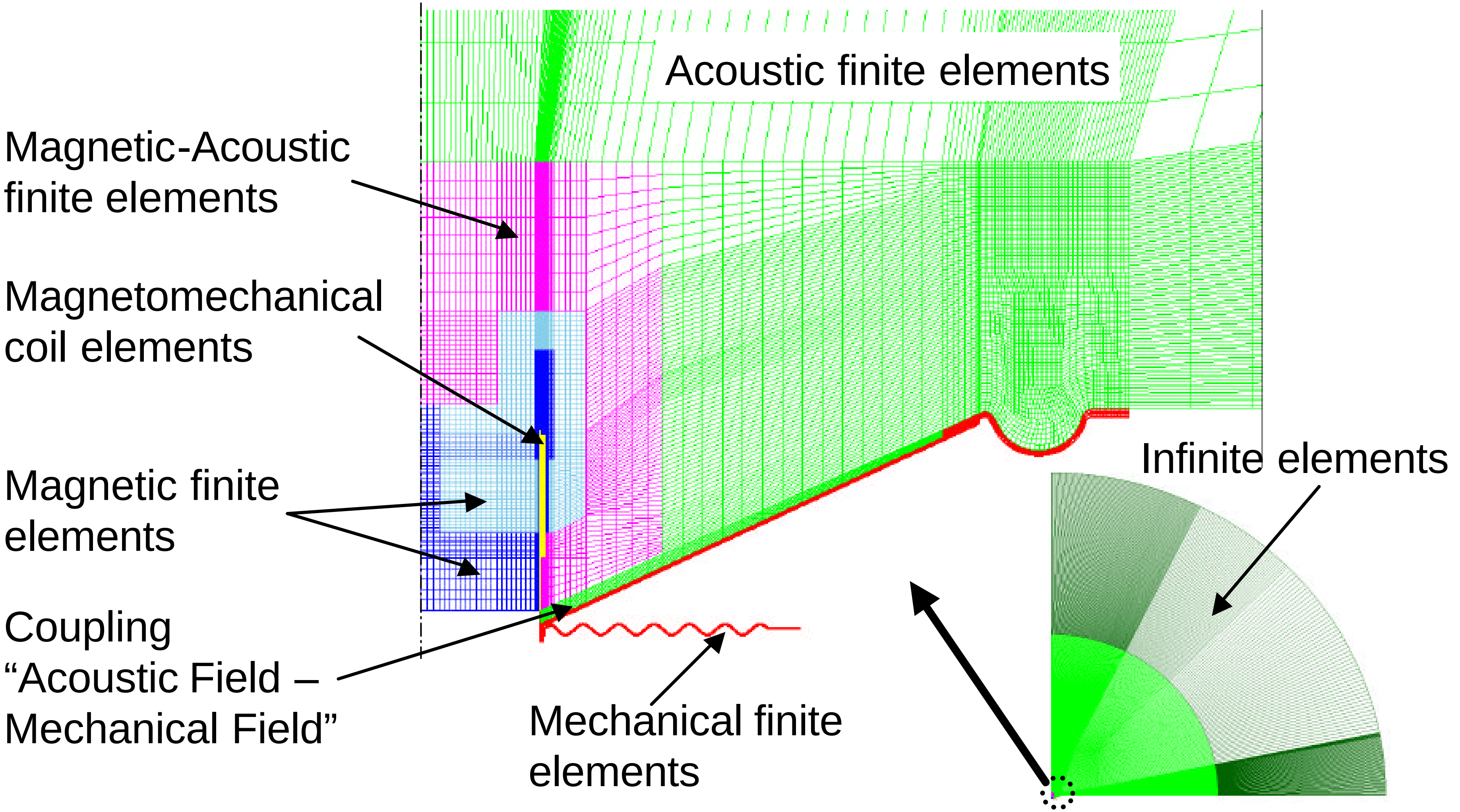
$$v_n = \vec{n} \cdot \left(\frac{\partial \vec{d}}{\partial t}\right) = -\vec{n} \cdot \nabla\psi = -\frac{\partial \psi}{\partial n}$$

Acoustic field

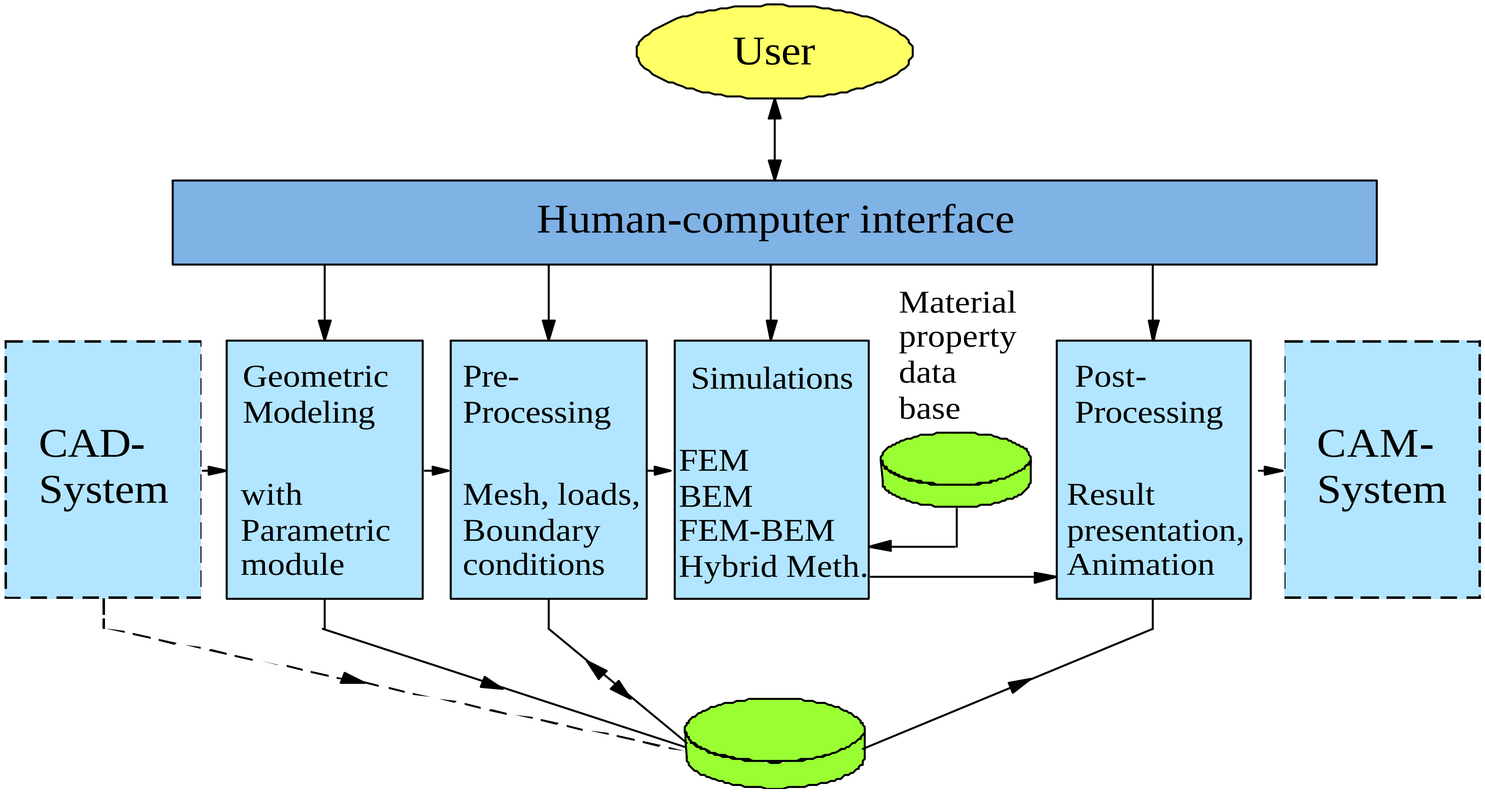
$$\nabla^2\psi = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} \qquad p = \rho\frac{\partial \psi}{\partial t}$$

$p(\vec{r}, t)$

FEM-Model of an Electrodynamic Loudspeaker



CAE Environment



Examples for Modeling Projects

- Medical Ultrasound Antennas
- High Voltage Quartz Sensors
- Ultrasonic Filling Level Sensor
- Ultrasonic Flowmeter
- Acoustic Power Source
- Micromachined Ultrasound Transducer
- Micromachined Pump
- Surface Acoustic Wave (SAW) Sensors
- Piezoelectric Stack Actuator
- Electrodynamic Loudspeaker
- Magnetic Valve
- Density Sensor
- Magnetic Angular Rate Sensor
- Magnetic Thickness Sensor

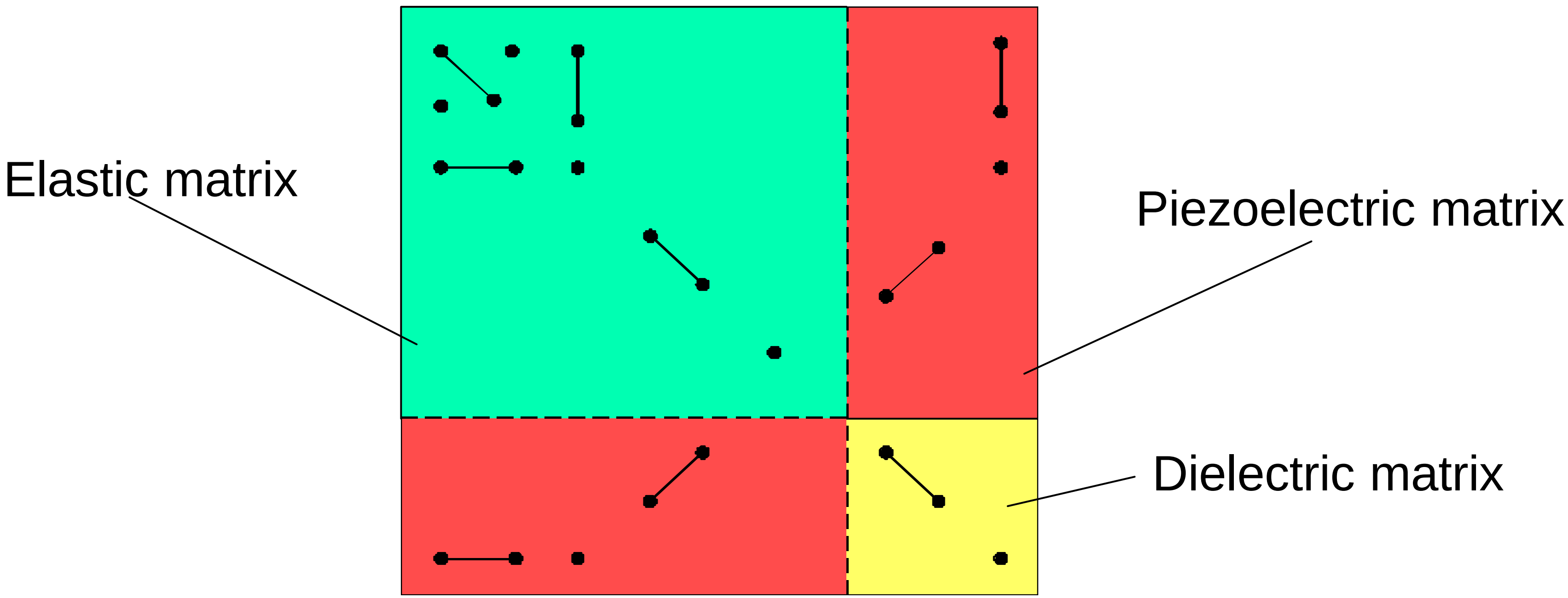
Problems which may arise in Transducer Modeling

- Abstraction / computer resources
- Material parameters
- Verification of results

Material Tensors of Piezoceramic Material

(class 6 mm)

Piezoelectric matrix of a piezoceramic



Material Tensors

(6mm class)

- Modulus of elasticity:

$$\mathbf{c}^E = \begin{pmatrix} c_{11}^E & c_{12}^E & c_{13}^E & 0 & 0 & 0 \\ c_{12}^E & c_{11}^E & c_{13}^E & 0 & 0 & 0 \\ c_{13}^E & c_{13}^E & c_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & \left(c_{11}^E - c_{12}^E\right)/2 \end{pmatrix}$$

- Piezoelectric modulus

$$\mathbf{e} = \begin{pmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{pmatrix}$$

- Dielectric modulus:

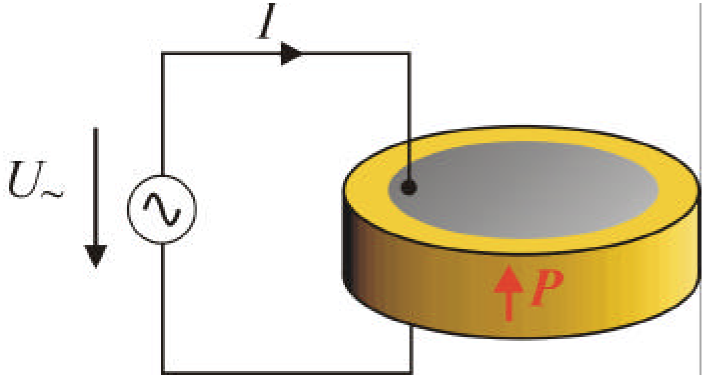
$$\varepsilon^S = \begin{pmatrix} \varepsilon_{11}^S & 0 & 0 \\ 0 & \varepsilon_{11}^S & 0 \\ 0 & 0 & \varepsilon_{33}^S \end{pmatrix}$$

10 material parameters

State of Art

- Test samples with special geometries:
  simplification to the one-dimensional case, direct relation between resonance frequencies and coefficients

Example: **thickness resonantor**



thickness << radius

$$\frac{k_t^2}{1-k_t^2} = \frac{e_{33}^2}{c_{33}^E \varepsilon_{33}^S}$$

$$k_t^2 = \frac{\pi}{2}\frac{f_s}{f_p}\tan\left(\frac{\pi}{2}\frac{f_p - f_s}{f_p}\right)$$

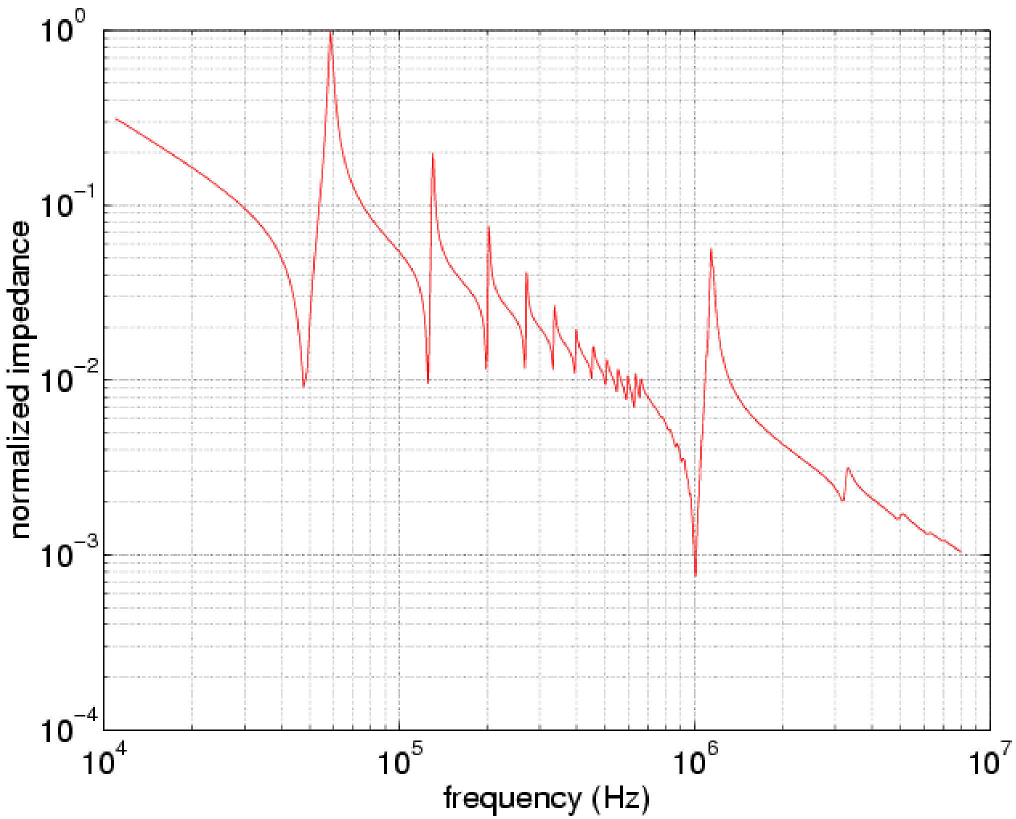
Identification by Simulation of the Full System

Find **material tensors**

$$\mathbf{c}^{\mathbf{E}}, \mathbf{e}, \boldsymbol{\varepsilon}^{\mathbf{S}}$$

from measured impedance

$$Z(\omega) = \frac{\widehat{\phi}(\omega)|_{\Gamma_e}}{\imath\omega\widehat{q}^e(\omega)}$$



Partial Differential Equation

- Partial differential equation (PDE):

$$
\begin{aligned}
-\omega^2\vec{u} - \mathbf{B}^T\left(\mathbf{c}^E\mathbf{B}\vec{u} + \mathbf{e}\,\mathrm{grad}\phi\right) &= 0 \\
-\mathrm{div}\left(\mathbf{e}\mathbf{B}\vec{u} - \boldsymbol{\varepsilon}^S\mathrm{grad}\phi\right) &= 0
\end{aligned}
$$

- Boundary condtions:

$$
\begin{aligned}
\sigma_{\mathrm{n}} &= 0 && \text{no surface stress} \\
\phi &= 0 && \text{grounded electrode} \\
\vec{D}\cdot\vec{n} &= -\frac{q^e}{A} && \text{loaded electrode} \\
\vec{D}\cdot\vec{n} &= 0 && \text{zero normal component}
\end{aligned}
$$

Algorithm

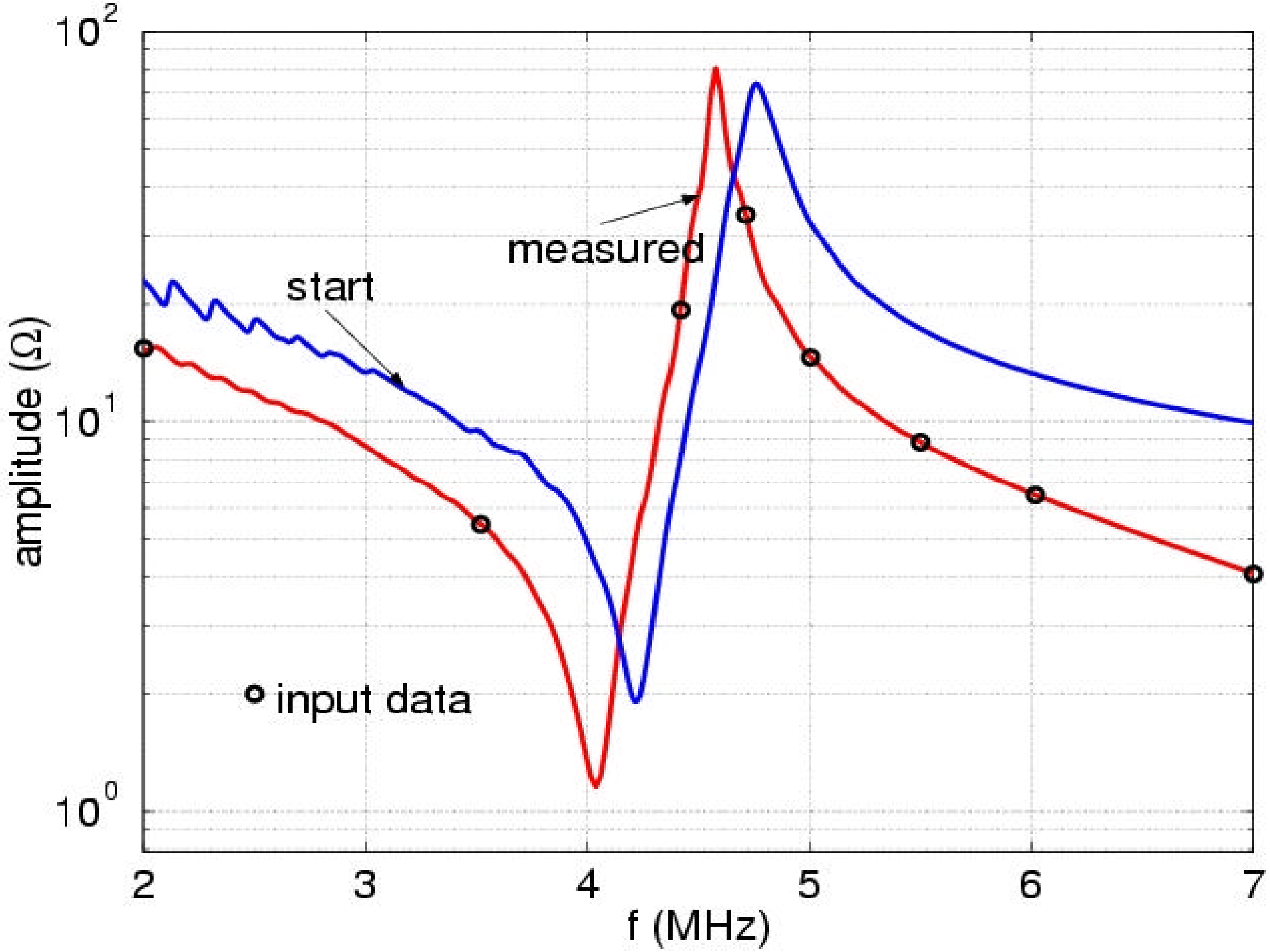
- **Input data:**
  - Measured amplitude and phase of the electric impedance as well as damping parameters at different frequencies

- **Iterative scheme based on**
  - Finite element simulation of the full PDE

    combined with

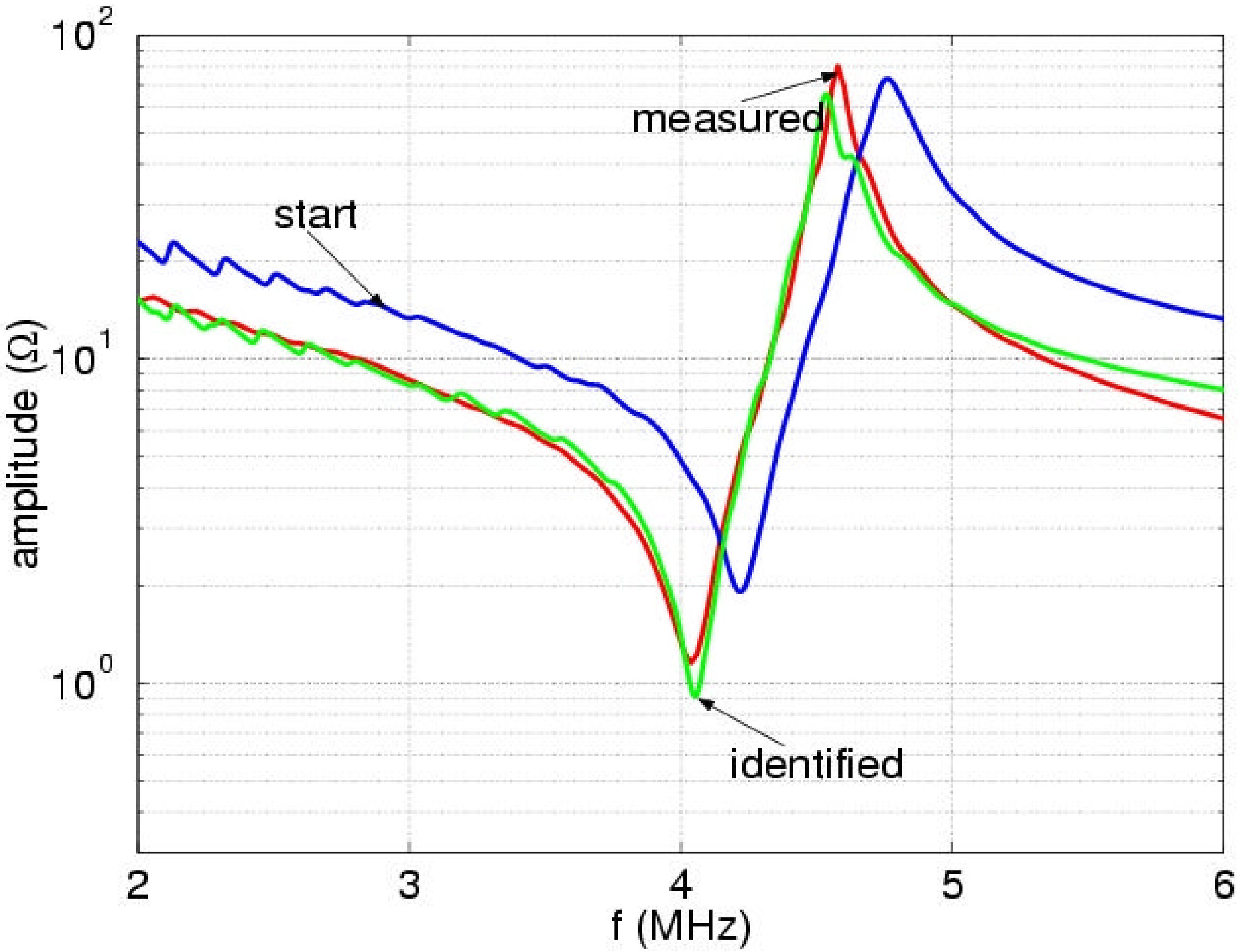
  - Newton-conjugate gradient inversion scheme
  - Stopping criterion: data noise level

Measured Electric Impedance



- Material: M1100
- Diameter: 11mm
- Thickness: 0.5mm
- Measurement: HP4194

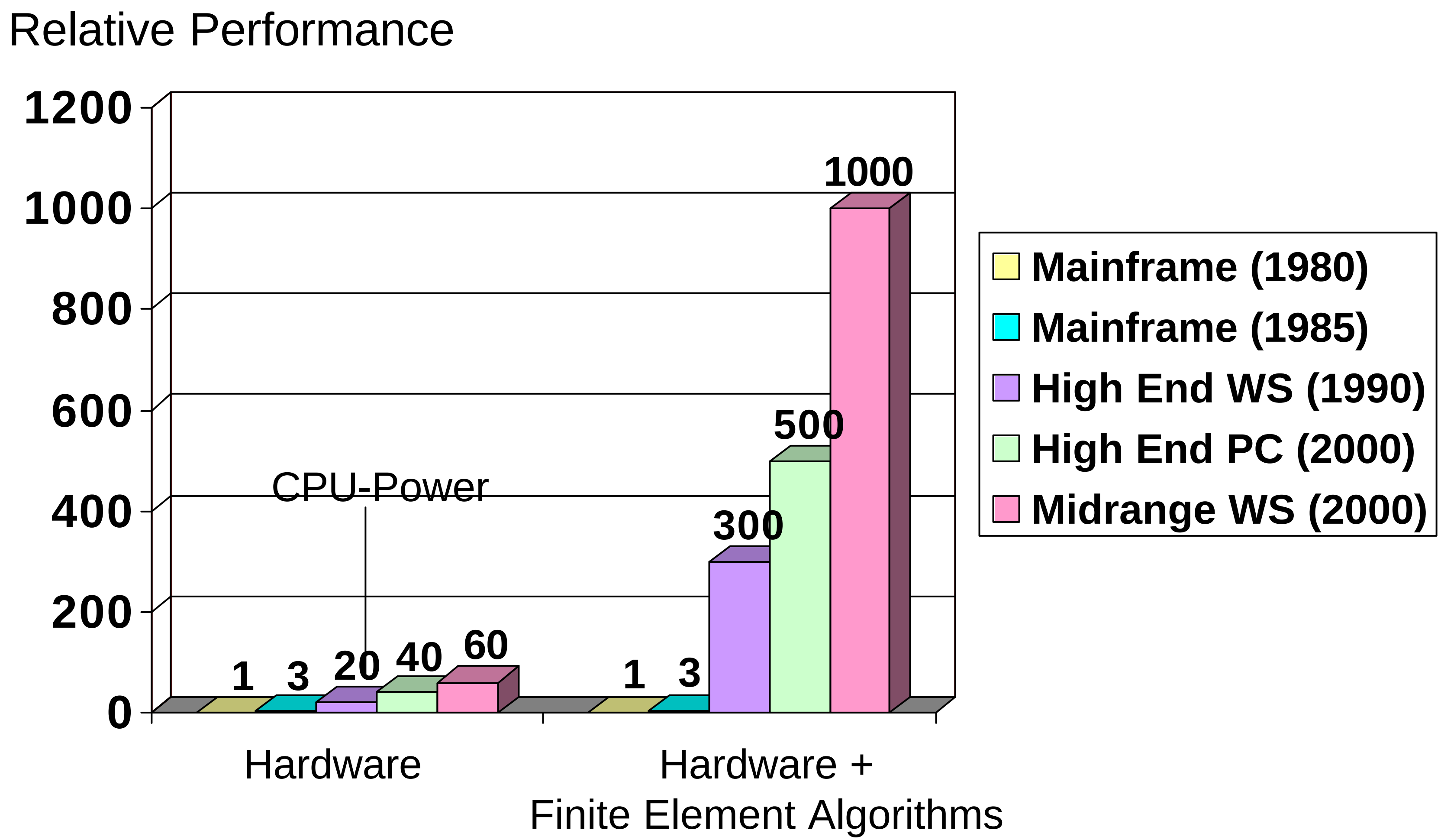
Result: Variation of all Parameters (I)



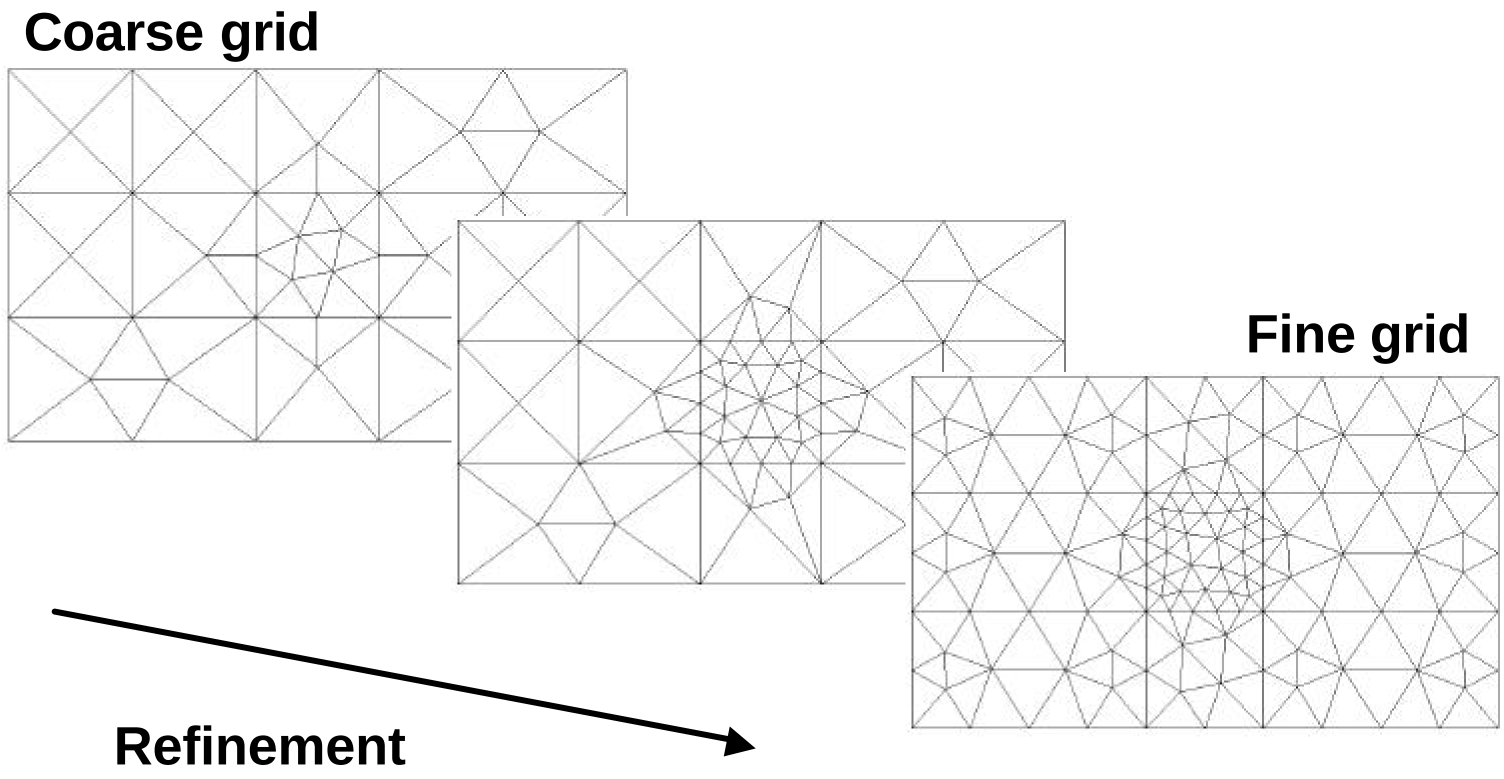
Result: Variation of all Parameters (II)

	start values	identified values	relative change (%)
c_{11}^E	1.433E+10	1.4061E+10	1.9
c_{33}^E	1.370E+11	1.2766E+11	6.8
c_{12}^E	8.611E+10	8.6116E+10	0.0
c_{13}^E	9.902E+10	1.0064E+11	1.6
c_{44}^E	2.340E+10	1.7636E+10	24.6
e_{15}	1.834E+01	1.4865E+01	18.9
e_{31}	-9.983E+00	-10.082E+00	1.0
e_{33}	2.454E+01	2.8805E+01	17.4
ε_{11}^S	1.5969E-08	1.70467E-08	6.7
ε_{33}^S	1.3523E-08	2.10551E-08	55.7

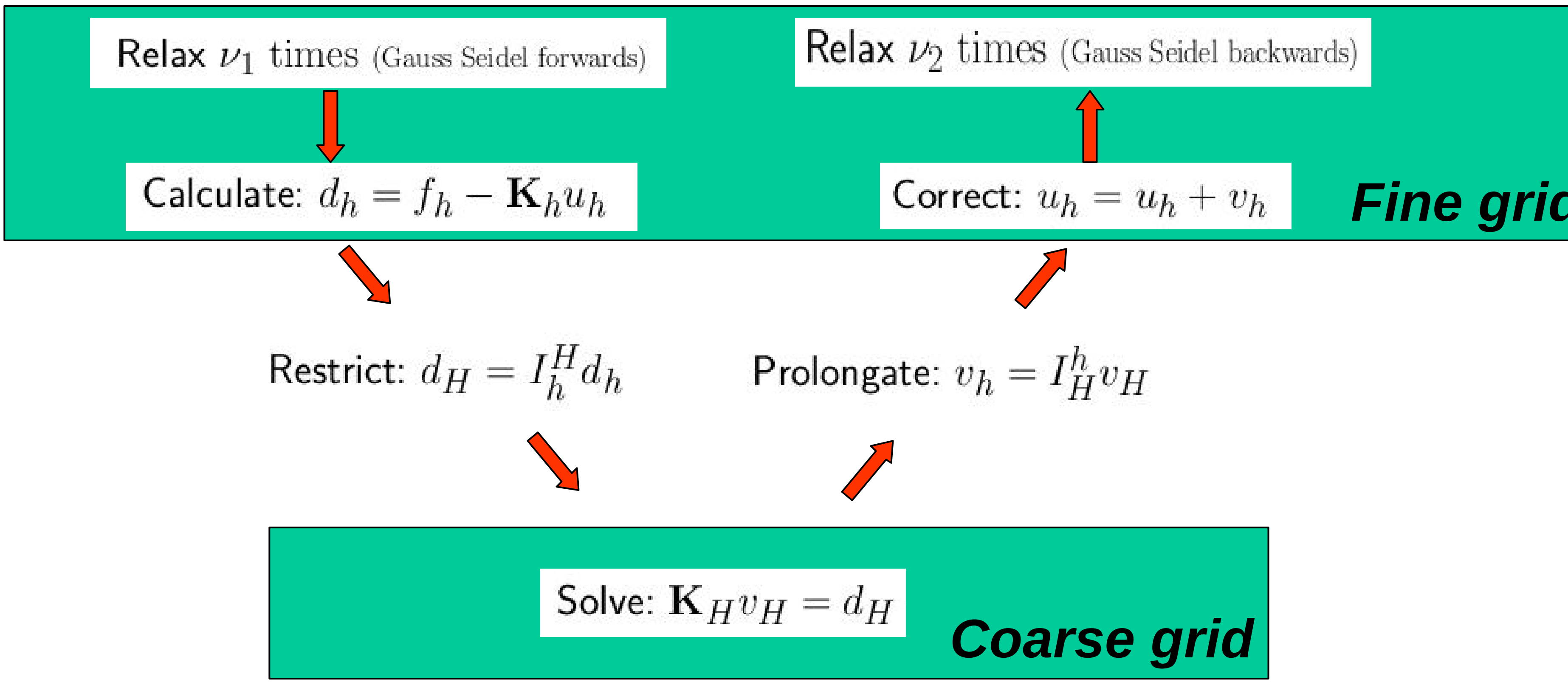
Increase in Computational Power over the last 20 Years



Multigrid Solution Strategy



Multigrid Method: Two Grid Algorithm

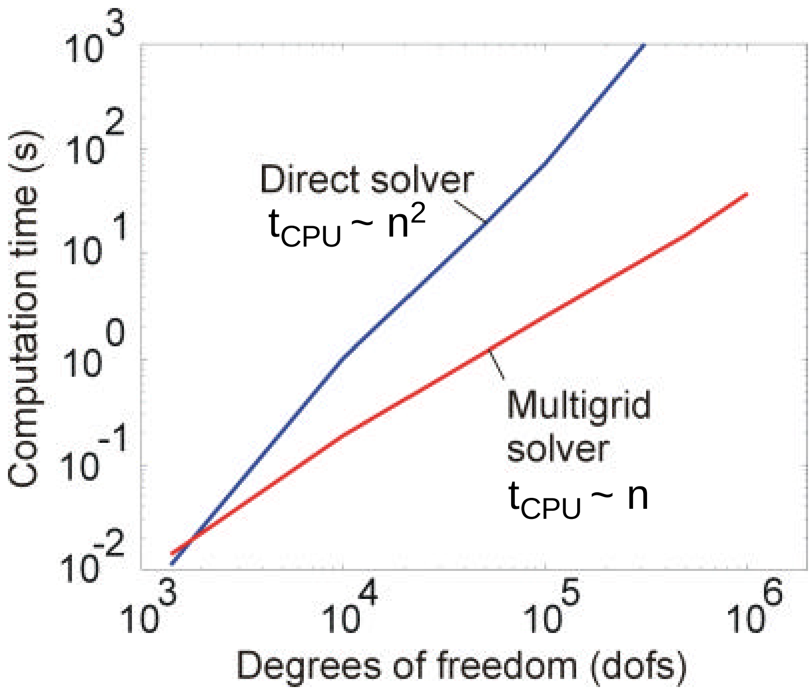


Multigrid Method: Motivation

Examing the error in the frequency domain:

- High frequency errors are well eliminated by few smoothing steps
- Once this is achieved, further smoothing steps results in less error improvement
- Transfer the solution to a coarser grid
- Low frequeny errors on the fine grid manifest themselves as high frequency errors on the coarse grid
- If the coarsest grid is reached, the equation is solved exactly.

Computation Times for Conventional and Multigrid Solvers

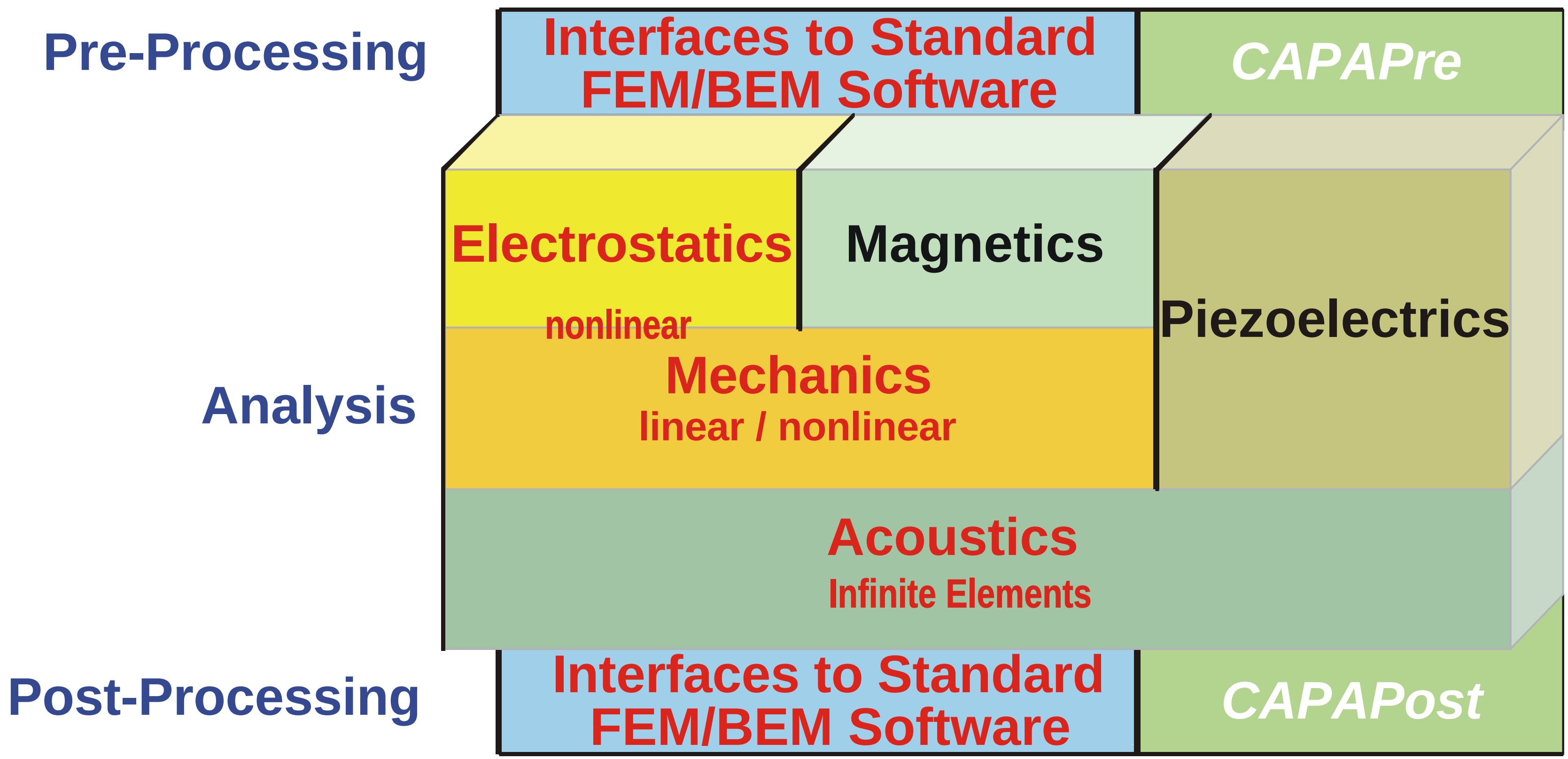


dofs (million)	Direct solver	Multigrid solver	Speed up factor
0.01	0.95 s	0.2 s	4.75
0.1	72 s	2.5 s	29
1	-	39 s	-

Selection of commercially available codes for coupled field problems (not complete)

- ABAQUS
- ADINA
- ANSYS
- ATILA
- CAPA
- FLUX 2D/3D
- NASTRAN
- NM-SESES
- PERMAS
- PZFLEX

CAPA Software System



CAD-System CAPA (I)
Numerical Methods

❑ Finite Element Method (FEM):	transducers, magnetic fields, electric fields, acoustic fields
❑ Boundary Element Method (BEM):	electric, magnetic and acoustic fields
❑ Coupled Methods (FEM/BEM):	multi-field problems
❑ Huygens/Kirchhoff-Programs:	acoustic fields

CAD-System CAPA (II)
Element Types

Piezoelectric elements:	mechanic, electrostatic, and piezoelectric problems
Magnetic elements:	magnetic and magneto-mechanic problems
Electrostatic elements:	electrostatic and coupled electrostatic-mechanic problems
Acoustic elements:	wave propagation in bounded and unbounded regions, fluid-structure interaction
Acoustic elements for media with flow:	wave propagation within flow in bounded and unbounded regions, fluid-structure interaction
Absorbing elements:	elements with adjustable absorbtion

CAD-System CAPA (III)
Analysis Types

Transient analysis:	time-domain, pulse-response, broad-band excitations
Harmonic analysis:	frequency domain, CW-excitation, narrow-band excitations
Eigenfrequencies:	calculation of eigenfrequencies and mode shapes

Acoustics

Linear acoustics

- Overview on numerical methods and algorithms
- Some things to consider
- Application: membrane sensitivity problem

Wave propagation in flowing media

- Comparison with standard acoustics
- Ultrasound flow meter

Nonlinear acoustics

- Finite element formulation
- Nonlinear plane wave radiation

Numerical Methods in Acoustics

Finite element method

- Applicable to transient problems (time domain) and harmonic problems (frequency domain)
- Open domains need special treatment

Boundary element method

- Efficiently applied only to harmonic problems
- Open domains can be easily included

Integral representations

- Based on Huygens/Kirchhoff integrals
- Sound pressure in any point of the domain determined by pressure and velocity on an enclosing surface
- Requires knowledge of pressure and velocity on enclosing surface

Numerical Methods in Acoustics

Wave equations

Transient

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

$\Downarrow$

Harmonic

$$\nabla^2\psi + k^2\psi = 0$$

Discrete System

$$\mathbf{M}\{\ddot{\Psi}\} + \mathbf{K}\{\Psi\} = \{F\}$$

FEM

$$\mathbf{K}^*\{\psi\} = \{R\}, \qquad \mathbf{K}^* = \mathbf{K} - \omega^2\mathbf{M}$$

BEM

$$\mathbf{H}\{\psi\} = \mathbf{G}\{\frac{\partial\psi}{\partial n}\}$$

Typical Problems in Acoustics

Sound radiation

- Encountered in transmit mode of transducers
- Fluid-solid interaction problems
- **Weak coupling**
  Presence of fluid medium does not effect behavior of radiating body
  May be solved by two separate simulations
- **Strong coupling**
  Ambient fluid medium strongly influences behavior of radiating body
  Requires fully coupled solution

Scattering of waves

- Disturbance of free wave propagation due to solid objects
- Hard vs. soft scatterers
  Soft objects neccessitate treatment of fluid-solid interaction

Transient Problems (I)

Time stepping procedures

- Spatial discretization by finite elements
- Temporal discretization by finite difference approximations
  Several well-known formulations available (Wilson, Newmark, α-Method)

Newmark algorithm

- Finite difference formulas

$$\psi_{n+1} = \psi_n + \Delta t \dot{\psi}_n + \frac{\Delta t^2}{2}\left[(1-2\beta)\,\ddot{\psi}_n + 2\beta\ddot{\psi}_{n+1}\right]$$

$$\dot{\psi}_{n+1} = \dot{\psi}_n + \Delta t\left[(1-\gamma)\,\ddot{\psi}_n + \gamma\ddot{\psi}_{n+1}\right]$$

- Effective mass matrix

$$\mathbf{M}^* = \mathbf{M} + \beta\Delta t^2\mathbf{K}$$

Transient Problems (II)

Newmark algorithm (cont.)

- Implicit system of equations

$$\mathbf{M}^*\{\ddot{\Psi}_{n+1}\} = \{F_{n+1}\} - \mathbf{K}\left[\{\Psi_n\} + \Delta t\{\dot{\Psi}_n\} + (1-2\beta)\frac{\Delta t^2}{2}\{\ddot{\Psi}_n\}\right]$$

- Use dedicated solvers for solution of equation system
  - ← direct, sparse solvers
    require factorization of matrix, stable, but need more memory
  - ← iterative solvers
    convergence strongly depends on mesh quality and pre-conditioner
    choice of best algorithm may be problem specific
- Explicit solution
  - ← Replace effective mass matrix by diagonal mass matrix
  - ← No solution of equation system required

Transient Problems (III)

Newmark algorithm (cont.)

- ❑ Implicit solution is unconditionally stable provided that
  $\gamma = 0.5$ and 2β ³ γ.
- ❑ Explicit solution poses upper limit on time step size (critical time step): time step must be smaller than transit time of wave for any element

Which solver to choose (some rules of thumb)?

- ❑ Small or medium size problems: implicit solution with direct solver (if memory plays no role)
- ❑ Medium size problems: implicit solution with iterative solver
  - ← symmetric, positive definite systems: conjugate gradient (CG)
  - ← unsymmetric, indefinite systems: generalized minimum residual (GMRES)
- ❑ Large scale problems: explicit solution

Harmonic and Eigenvalue Problems

Harmonic problems

- Working in frequency domain with complex system of equations
  $\leftarrow$ Increase in memory demands by a factor of 2
- Explicit solution not available $\leftarrow$ direct or iterative solvers
  However: choice of iterative solver is even more sensible

Eigenvalue problems

- Iterative solution algorithms
- Subspace iteration algorithm (K. J. Bathe)
  Original eigenvalue problem is projected onto a subspace of much smaller dimension
- Lanczos algorithm
  Eigenvalue problem is solved by approximation with a subspace of increasing dimension (Krylov-type subspace)

Numerical Effects in Transient Problems

- Period elongation
  Depends on ratio of time step and period of signal
  Significant, if less than 20 samples per period and large propagation distances

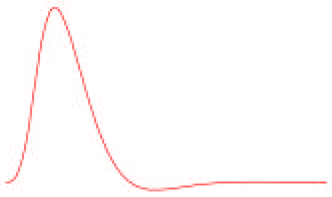
- Algorithmic damping
  Depends only on selection of integration parameters
  Does not show up in Newmark algorithm with standard parameters
  (β = 0.25 and γ = 0.5) but is always present if $\gamma > 0.5$

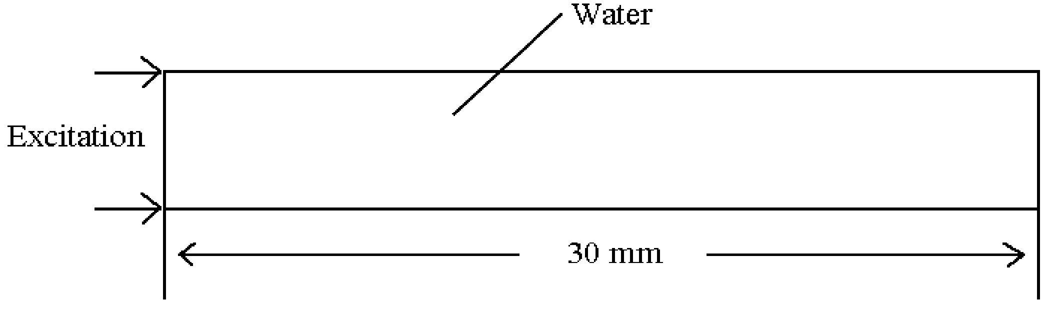
- Numerical Dispersion
  Depends on the combination of element and time step size
  May result in faster transit times than expected due to velocity of sound

Example: Plane Wave Propagation (I)

- Objective
  Study influence of time step and element size for a simple, onedimensional transient wave propagation problem.

- Setup
  Single row of acoustic finite elements
  Boundary conditions: rigid walls
  Excitation: spike with large bandwith (-6 dB 1.8 MHz, -20 dB 3.8 MHz)



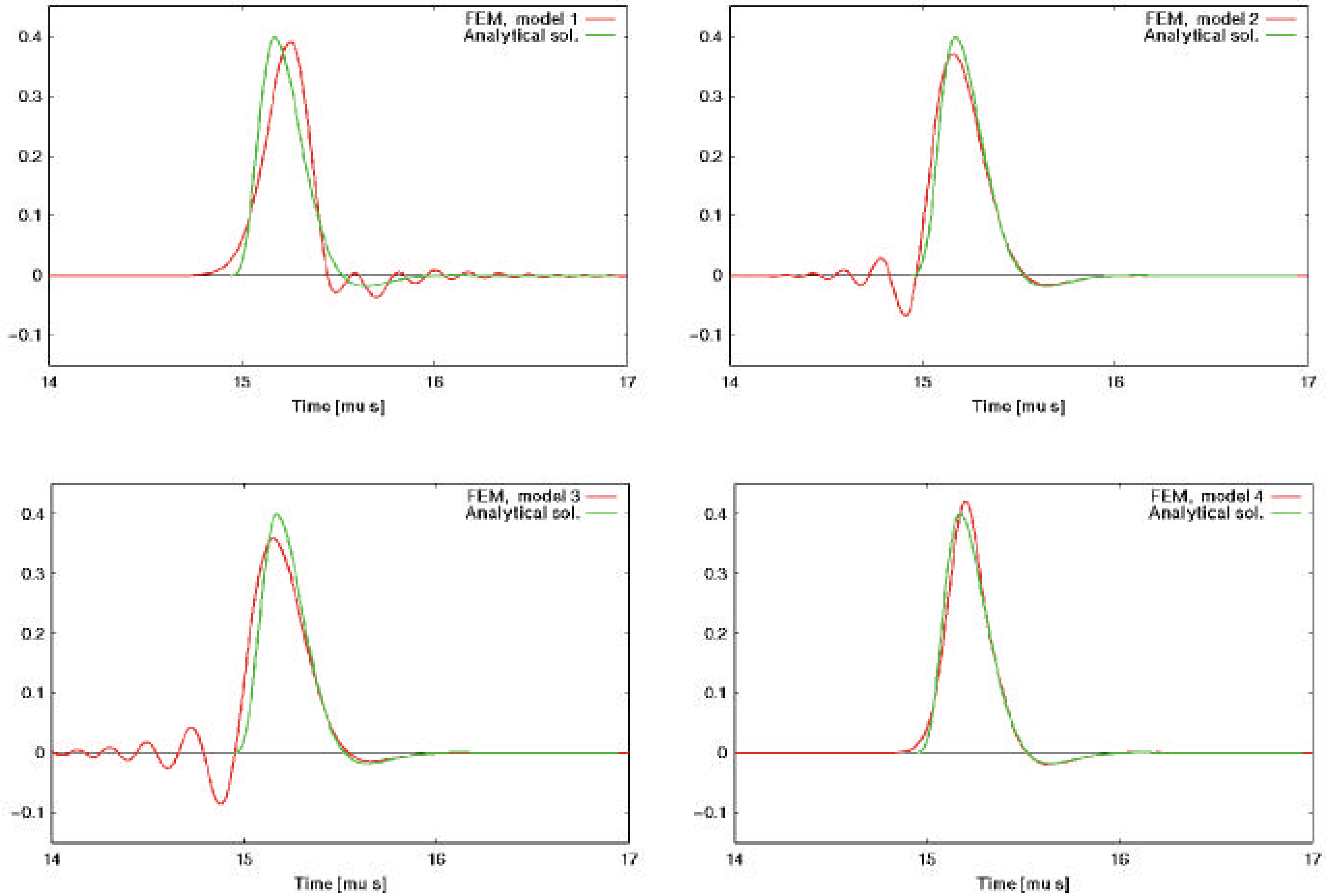


Example: Plane Wave Propagation (II)

- Discretization parameters

Model	Element size	Time step
1	30.0 mm	20 ns
2	30.0 mm	10 ns
3	30.0 mm	5 ns
4	15.0 mm	10 ns
5	7.50 mm	5 ns

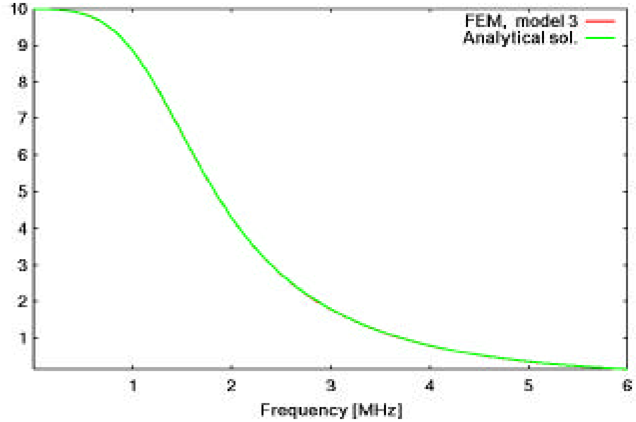
Example: Plane Wave Propagation (III)

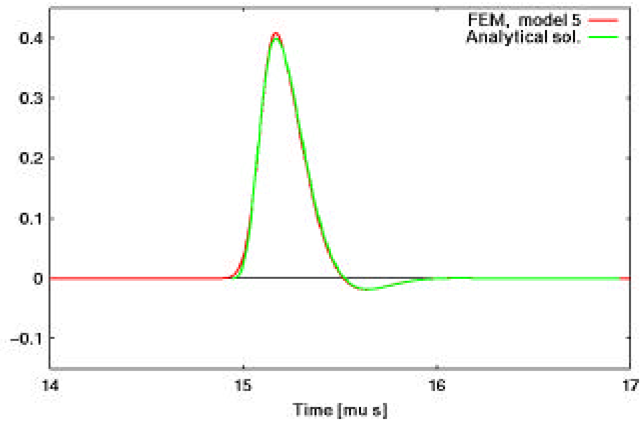


Example: Plane Wave Propagation (IV)

Conclusion

- Mismatch between time step and spatial discretization (models 1-3)
  ← Numerical dispersion
- Numerical dispersion is limited to the time domain and vanishes for fine discretizations





- For every spatial discretization, an optimal time step minimizing numerical dispersion may be chosen

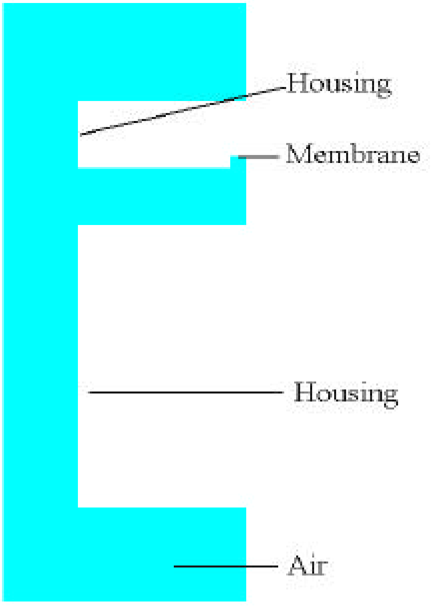
Sensitivity of Membrane Structure (I)

Problem

Study the increase in sensitivity of an axisymmetric membrane structure (microphone) which has been detected in measurements

Approach

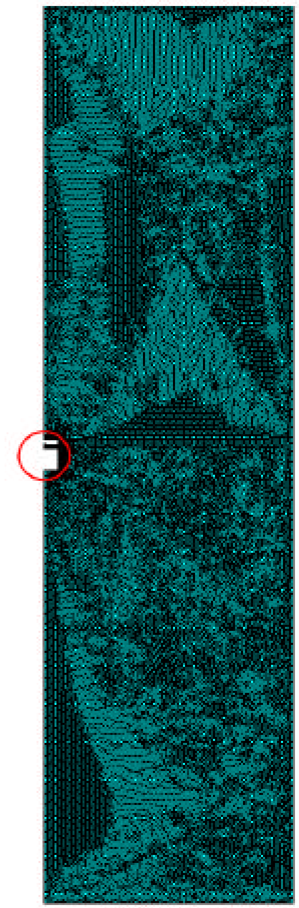
- Generate a finite element model of membrane including housing and surrounding air
- Excite a plane wave with broadband spectrum on top of the model
- Compare the pressure signal (spectrum) at the membrane surface with excitation signal



Sensitivity of Membrane Structure (II)

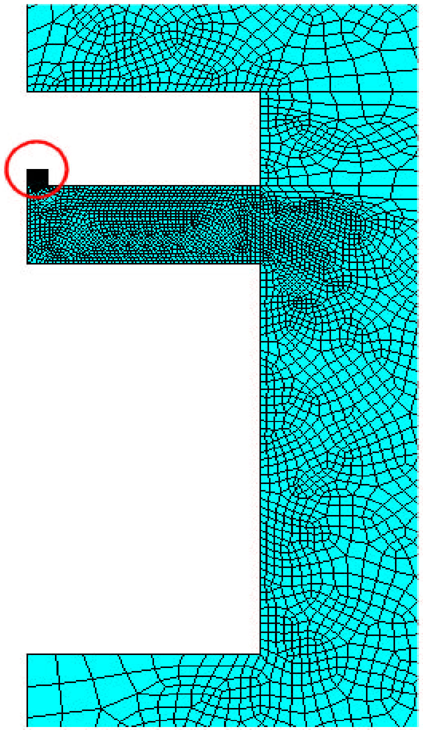
Problems encountered in modeling

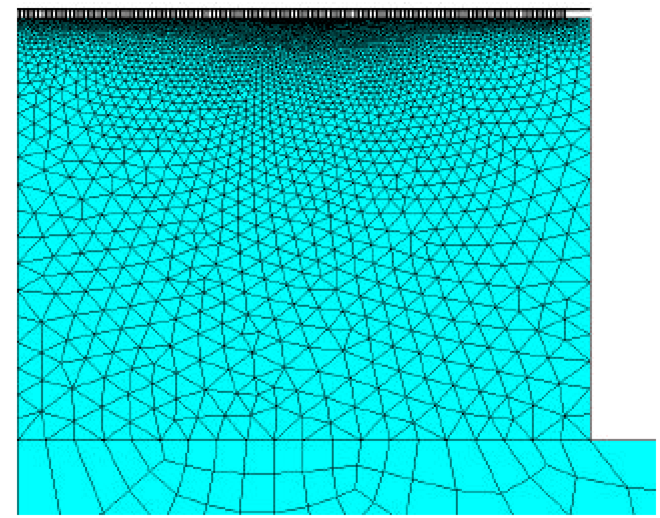
- Dimensions of membrane (typ. 1 μm) very small compared with wavelength (typ. 1 – 30 cm)
  ←explicit solver ruled out
- Transient approach requires reflection free time-window near membrane which increases model size
  ← direct solver ruled out
- Large model size (50 cm) necessitates smooth transition from very small to larger elements



Finite Element Model

Sensitivity of Membrane Structure (III)



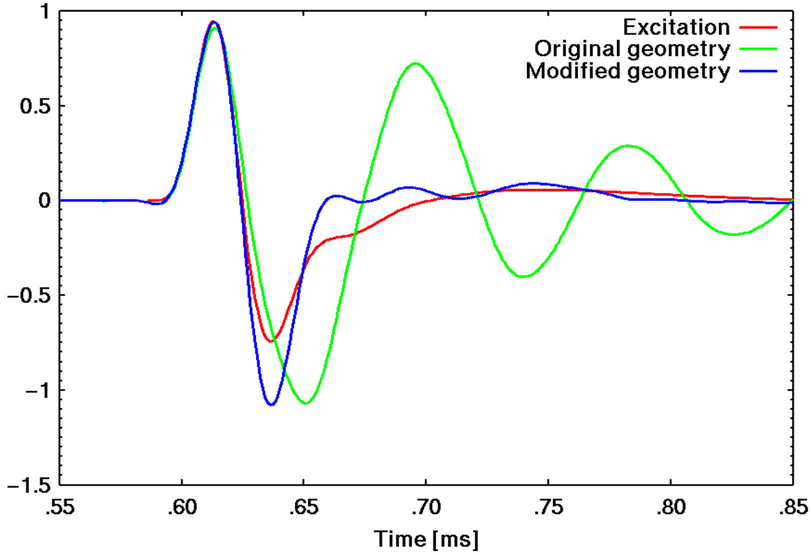


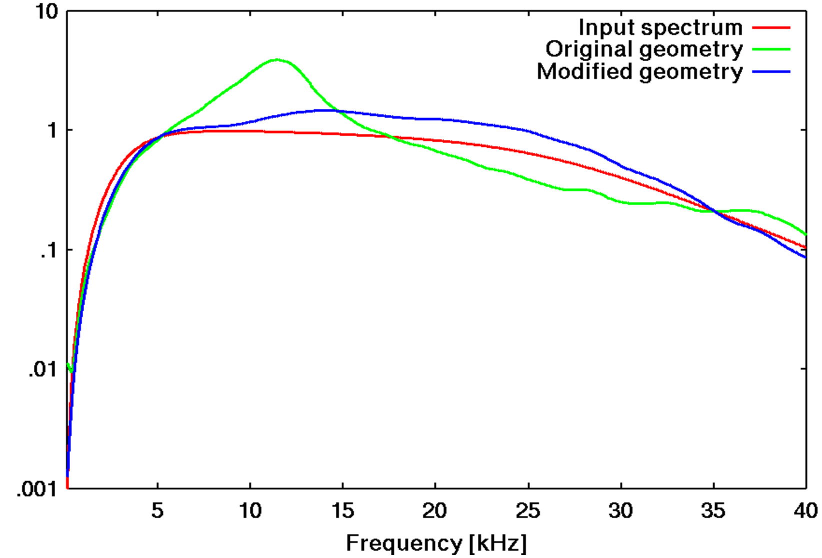
Some details of finite element grid

Sensitivity of Membrane Structure (IV)

Simulation results:

The frequency dependency of the membrane sensitivity results from resonances due to the backing volume and the housing of the membrane. Geometry modifications successfully eliminated this resonatic behavior.





Pressure signals and corresponding spectra for original and modified geometry

Wave Propagation in Flowing Media

- Standard wave equations limited to wave propagation in media at rest
- Many applications, like flowmeters, require numerical methods for waves propagating in flowing media
- Modified wave equation by Pierce

$$\nabla \cdot (\rho \nabla \psi) - \rho D_t \left(\frac{1}{c^2} D_t \psi \right) = 0$$

The differential operator

$$D_t = \frac{\partial}{\partial t} + v \cdot \nabla$$

describes the time derivative following the ambient flow.
Generalized velocity potential ψ related to pressure and velocity by means of

$$p = \rho \cdot D_t \psi, \qquad v = -\nabla \psi$$

- Covers only applications, in which flow is not disturbed by acoustic wave

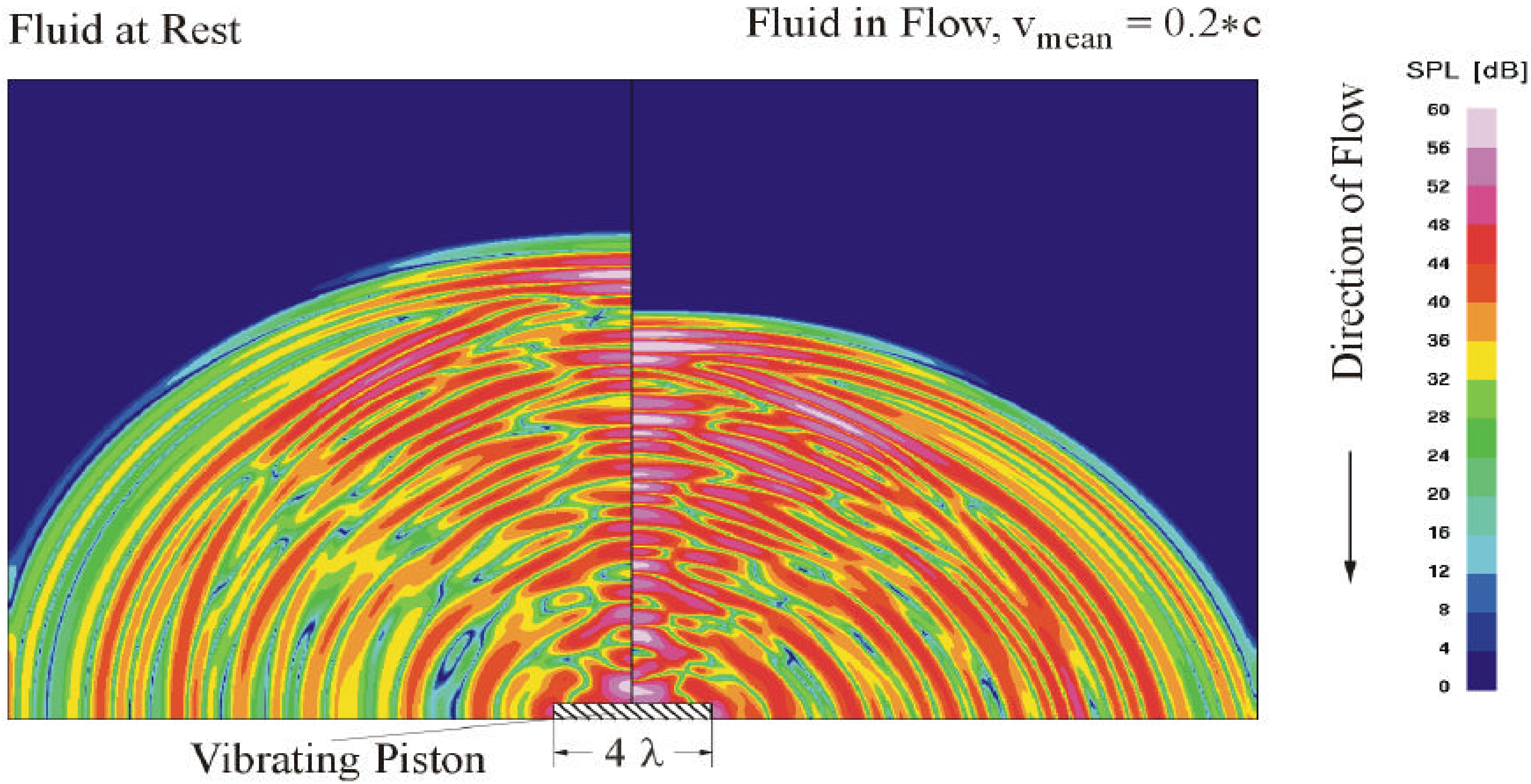
Finite Element Method for Wave Propagation in Flowing Media

- Finite element discretization applied to the weak form of Pierce's differential equation results in

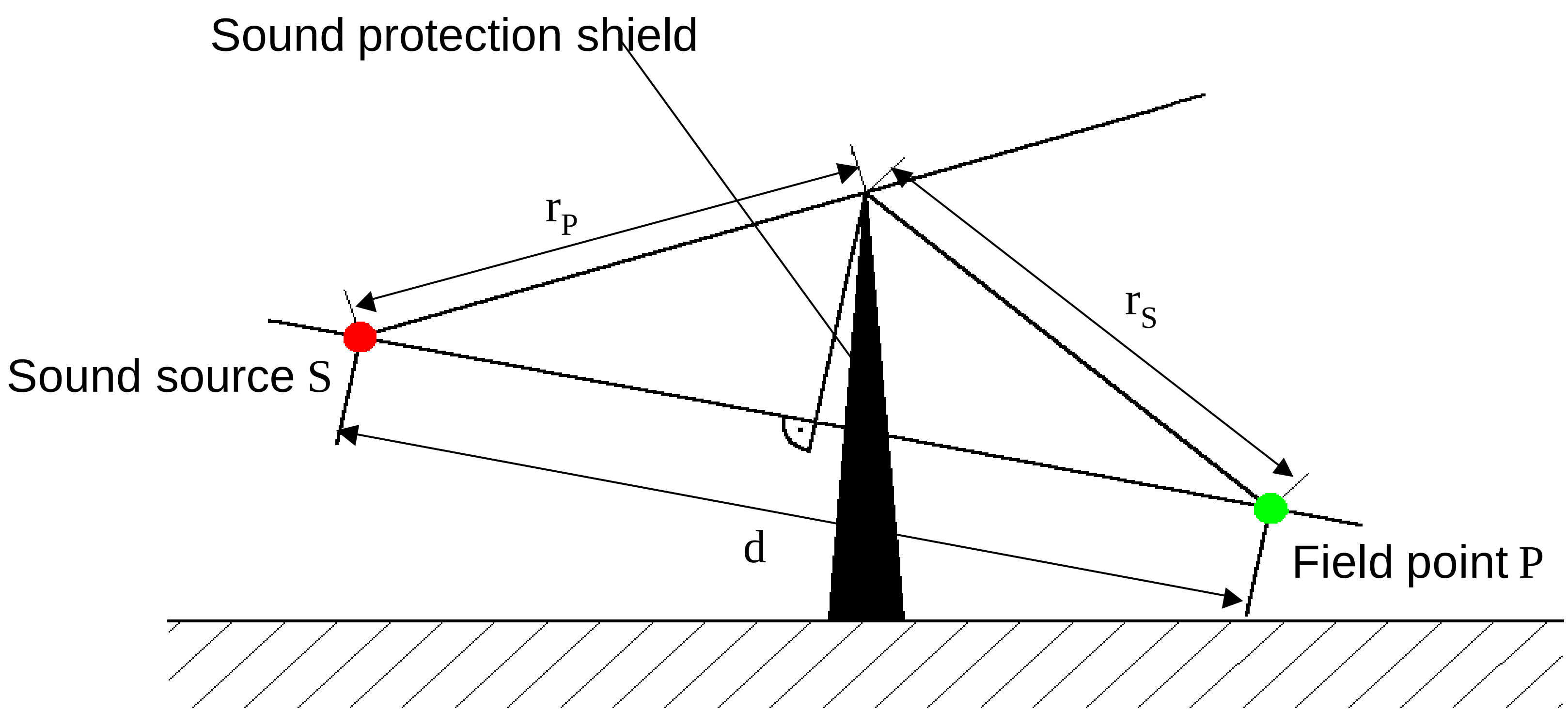
$$\mathbf{K}\{\Psi\} + \mathbf{C}\{\dot{\Psi}\} + \mathbf{M}\{\ddot{\Psi}\} = \{F\}$$

- Comparison with the standard system
  - Even for undamped systems, a *damping matrix* is present
  - Element and system matrices are no longer symmetric
    - ← higher demands regarding memory and computational efforts
    - ← standard CG-algorithm not applicable
- Extensions to infinite elements and fluid-structure coupling very similar to standard acoustics
- Due to weak coupling with flow, separate simulations for flow and acoustic wave propagation are feasible

Radiating Piston in a Tube

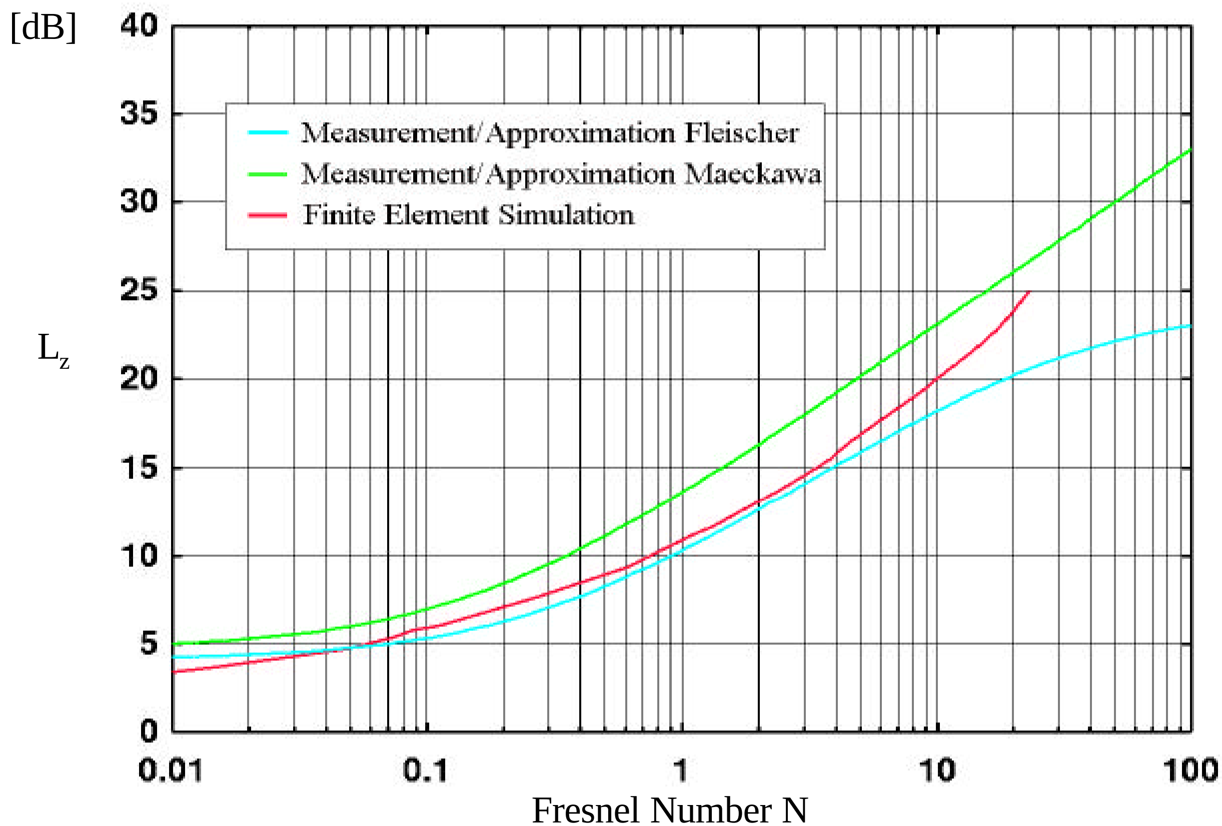


Sound Protection Shield: Geometry Setup



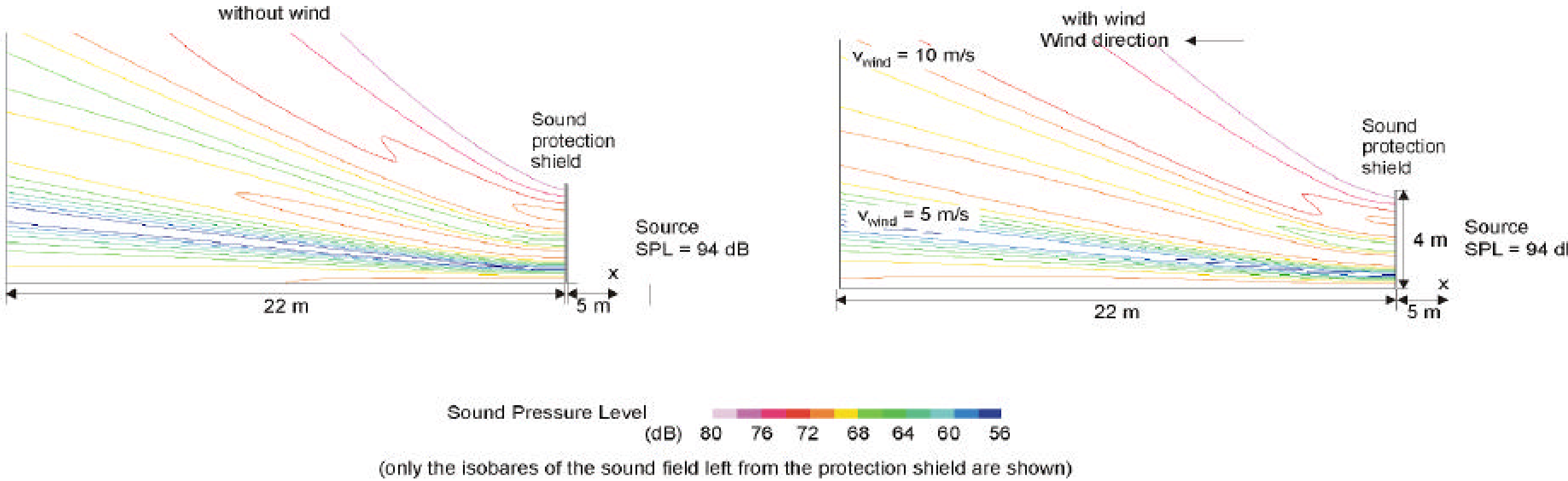
- Fresnel number $N = (r_P + r_S - d) / (\lambda/2)$

Sound Protection Shield: Insertion Loss



Comparison of calculated insertion loss L_z with approximations

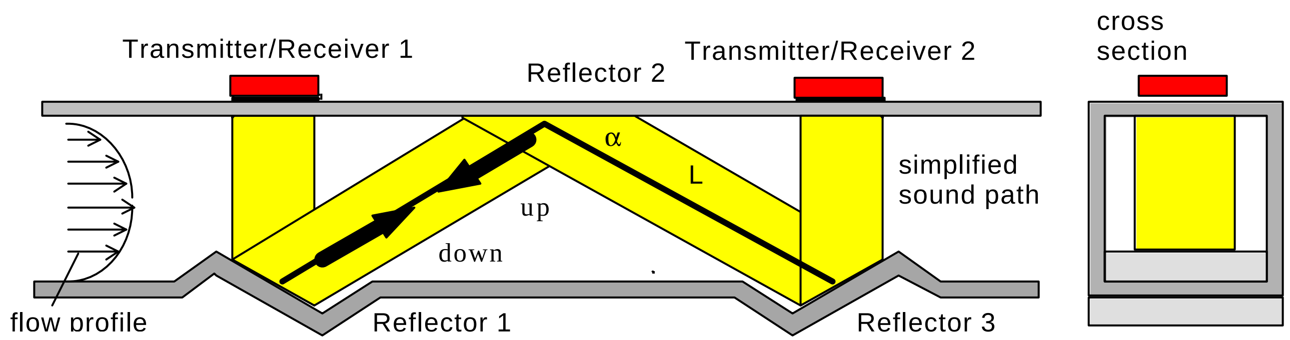
Influence of Wind Effects on Insertion Loss



- Wind effects may reduce insertion loss of the sound protection shield
- An increase in sound pressure level of 3 dB and more is observed

Finite Element Simulation of Ultrasound Flowmeters

Principle



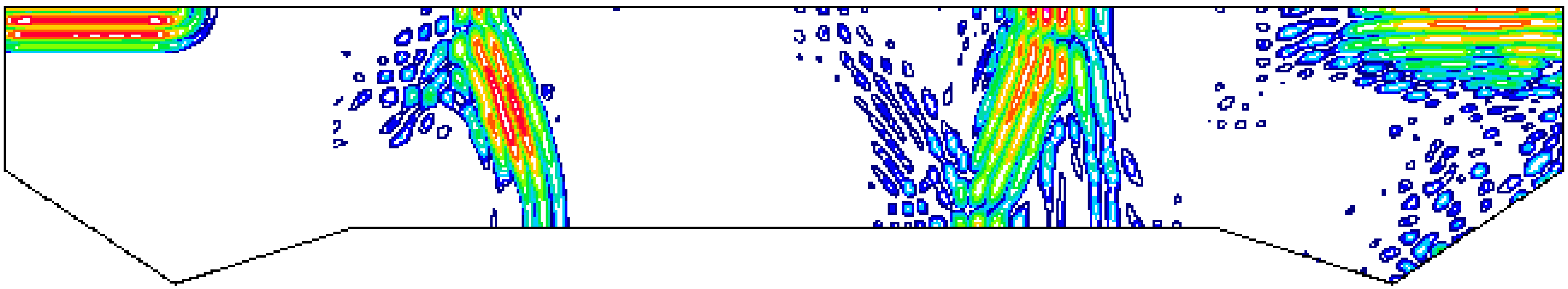
Finite Element Model

- 2D simulation, flowing media: water
- Approx. 800.000 finite elements and 6000 time steps
- Element size ranges from 12-30 elements per wavelength
- Care must be taken regarding element size near reflectors (critical region)

Simulation of Ultrasound Flowmeters

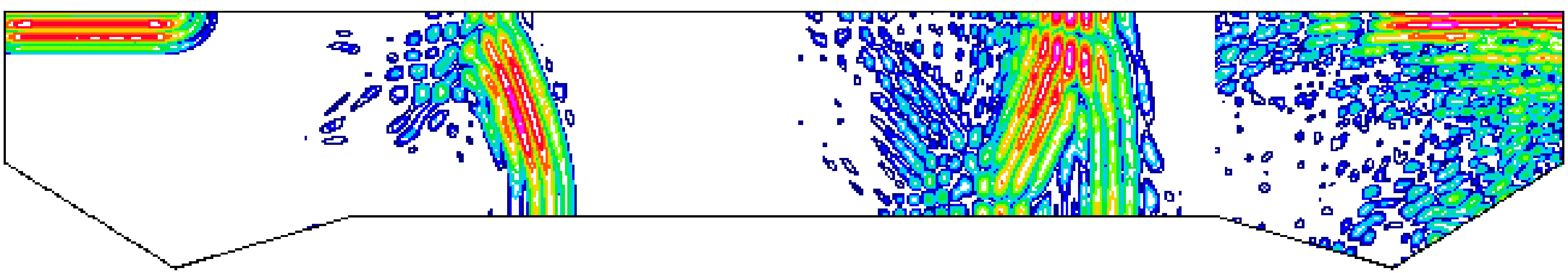
- Media at rest

t = 6 ms t = 48 ms t = 84 ms t = 128 ms



- Turbulent flow from left to right, mean velocity 30 m/s

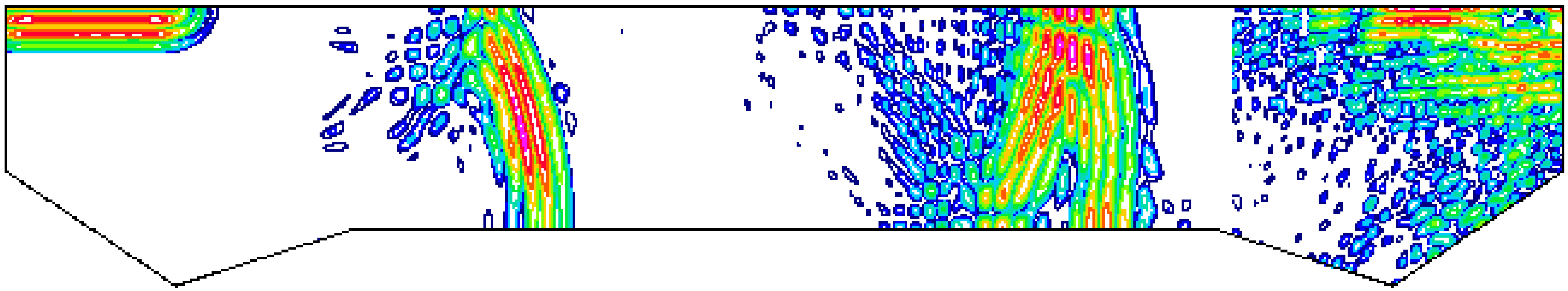
t = 6 ms t = 48 ms t = 84 ms t = 128 ms



Simulation of Ultrasound Flowmeters

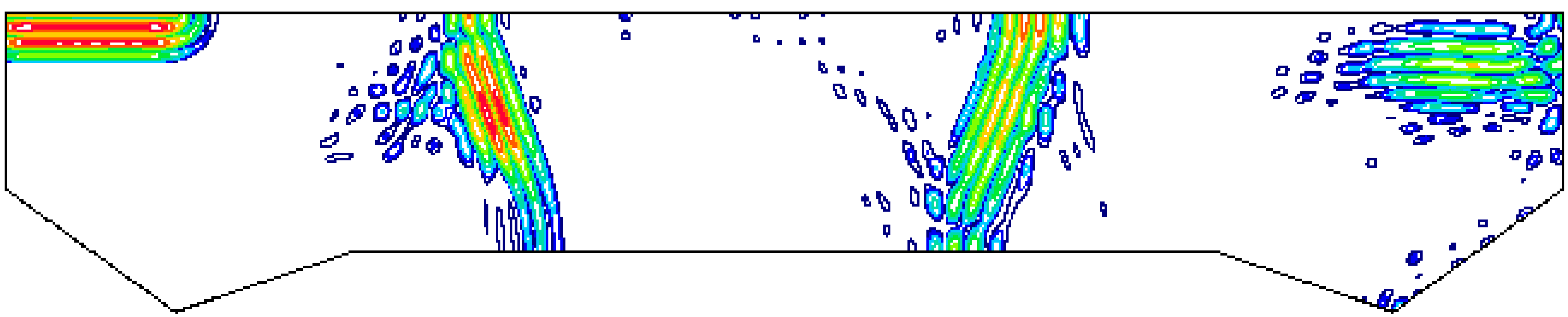
- Laminar flow from left to right, mean velocity 34 m/s

t = 6 ms t = 48 ms t = 84 ms t = 128 ms

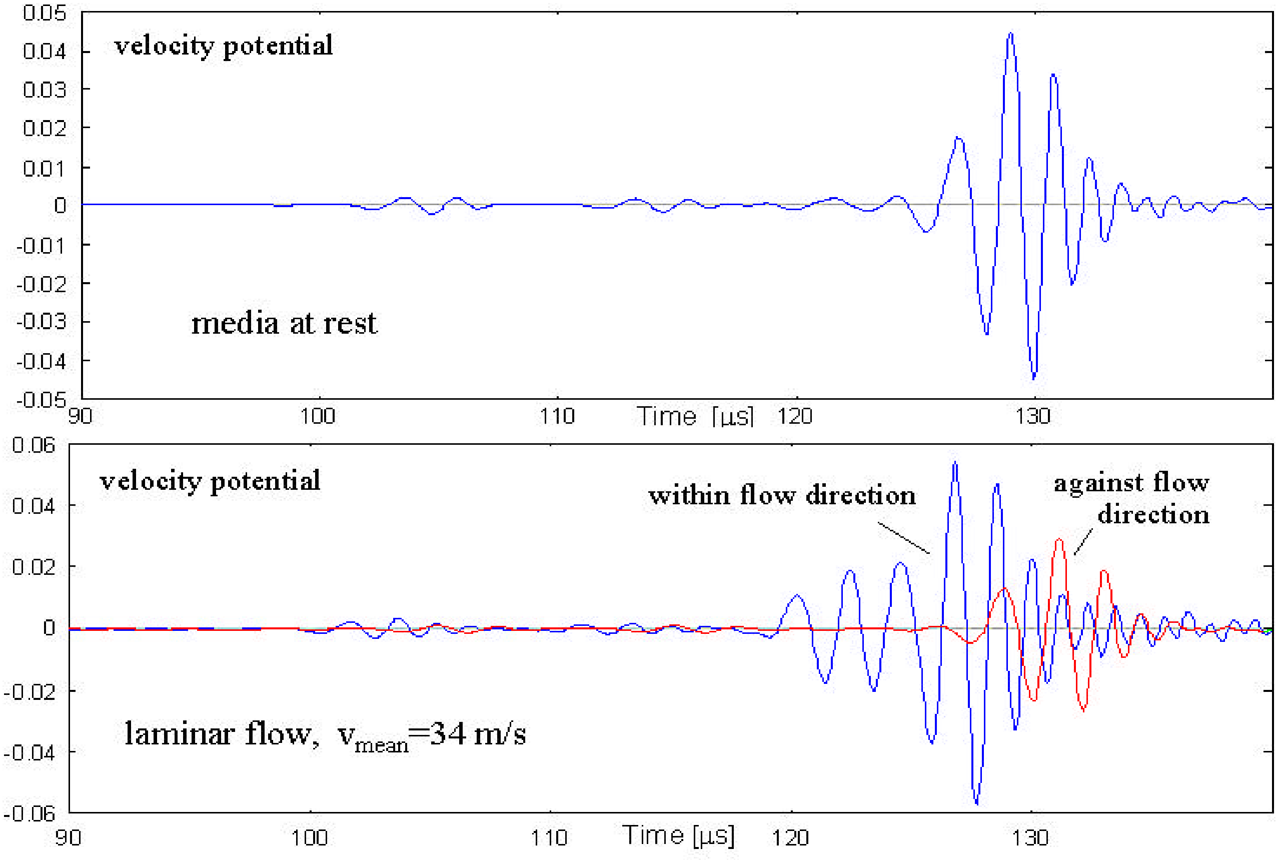


- Laminar flow from right to left, mean velocity 34 m/s

t = 6 ms t = 48 ms t = 84 ms t = 128 ms



Simulation of Ultrasound Flowmeters



Time signals in front of receiver

Nonlinear Acoustics

- Standard wave equations only cover linear acoustics
- High power applications, like High Intensive Focussed Ultrasound (HIFU), however, exhibit strong nonlinear effects
- Nonlinear wave equation derived by Kuznetsov

$$\frac{1}{c_0{}^2}\frac{\partial^2\psi}{\partial t^2} - \Delta\psi = \frac{1}{c_0{}^2}\frac{\partial}{\partial t}\left(b(\triangle\psi) + \frac{B/A}{2c_0{}^2}(\frac{\partial\psi}{\partial t})^2 + (\nabla\psi)^2\right)$$

  B/A denotes the nonlinearity parameter of the fluid and *b* the damping coefficient

- General formulation applicable to arbitrary 3D problems
- Generation of higher harmonics
- Formation of weak shocks
- Dissipation

Finite Element Method for NonlinearAcoustics

- Finite element discretization applied to weak form of Kuznetsov's nonlinear wave equation leads to discrete system

$$\mathbf{K}\{\psi\} + \mathbf{C}\{\dot{\psi}\} + \mathbf{M}\{\ddot{\psi}\} = \mathbf{N_1}(\dot{\psi})\{\ddot{\psi}\} + \mathbf{N_2}(\psi)\{\dot{\psi}\}$$

- Left side of equation system equivalent to standard acoustic finite element formulation (only linear terms)
- All nonlinearities are summarized on the right hand side
  - No reformulation/refactorization of system matrices needed
  - Efficient solution algorithm
- Reduced integration may be used to calculate the nonlinear parts of the right hand side
- Fixed-point iteration algorithm converges within a few number of iterations

Nonlinear Plane Wave Problem

- Onedimensional problem with analytical solutions available
- Shock formation distance

$$\sigma = \left(\rho_0 c_0^3\right) / \left[\left(1 + \frac{B}{2A}\right) w p_0\right]$$

- p_0 and w amplitude and period of the driving pressure excitation
- Fubini solution

$$p(x) = \sum_{n=1}^{\infty} \frac{2\sigma}{nx} J_n\left(n\frac{x}{\sigma}\right) \sin\left(n(\omega t - kx)\right)$$

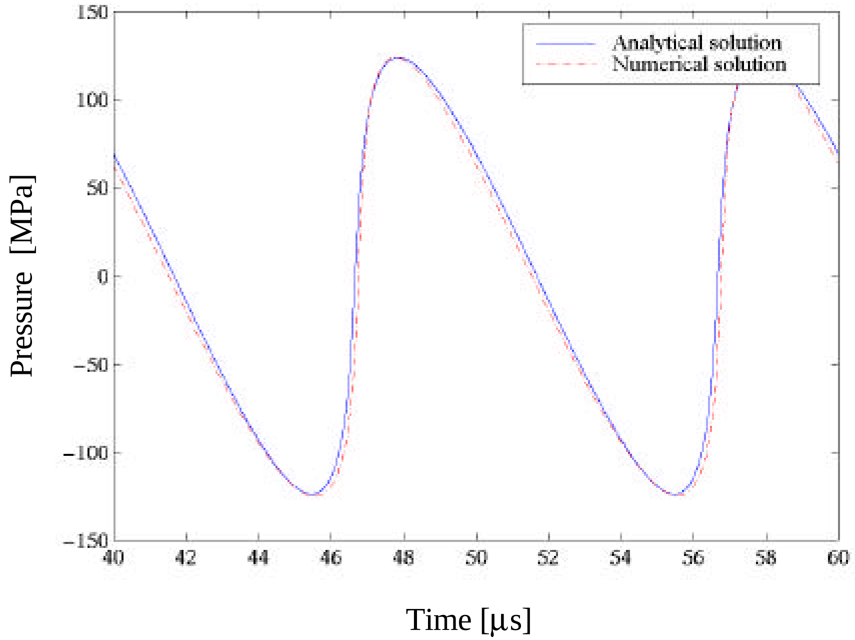
valid in the distance range up to x = σ (J_n)denotes the n-th Bessel function)

- Fay solution

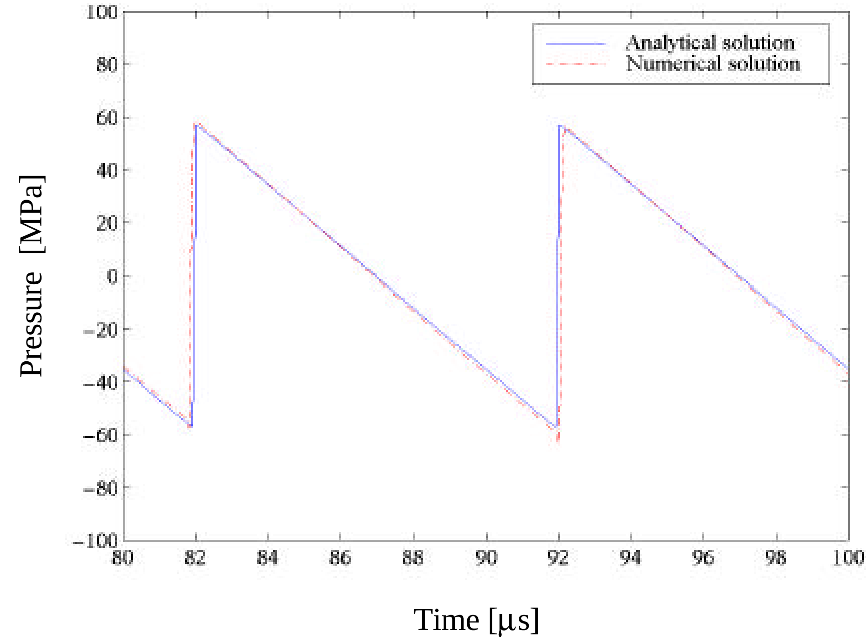
$$p(x) = \sum_{n=1}^{\infty} \frac{2/\Gamma}{\sinh(n(1+\frac{x}{\sigma})/\Gamma)} \sin\left(n(\omega t - kx)\right)$$

valid in the distange range from x = 3σ up to x = 5σ

Nonlinear Plane Wave Simulation

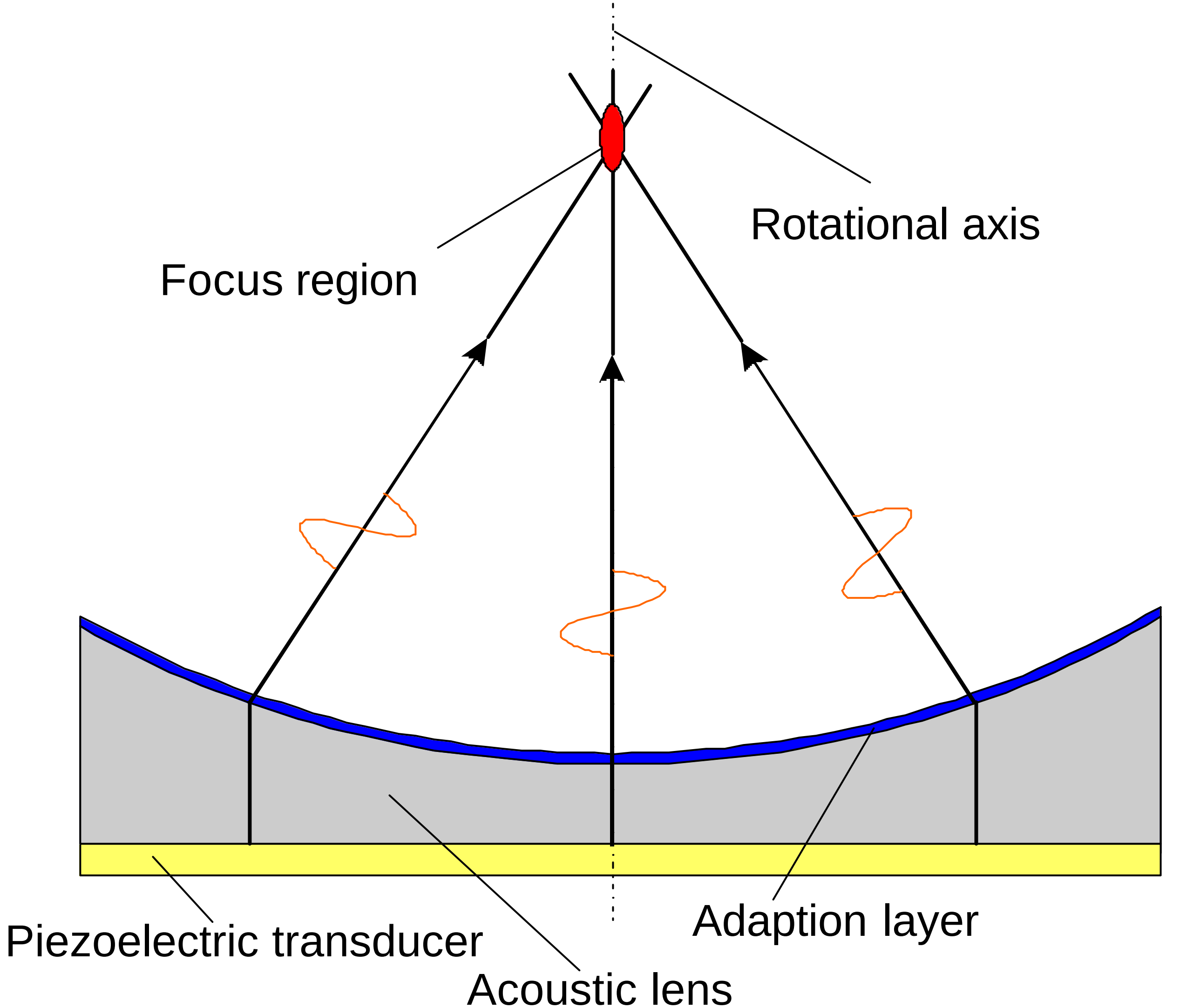


Comparison of numerical results with Fubini solution at x = σ

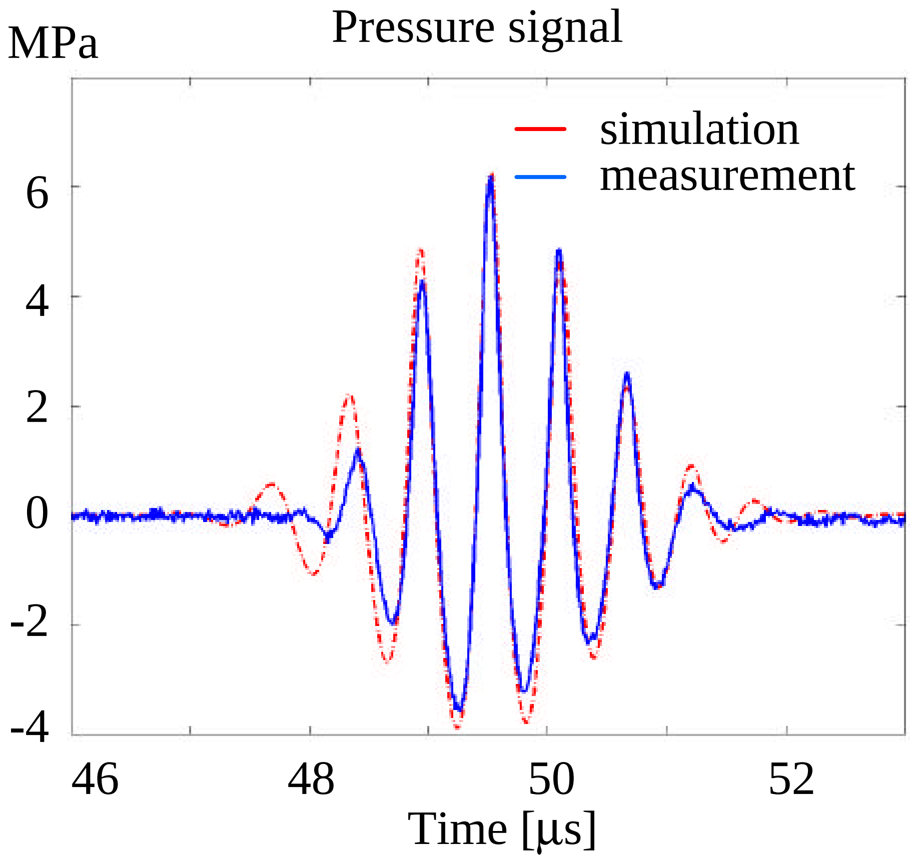


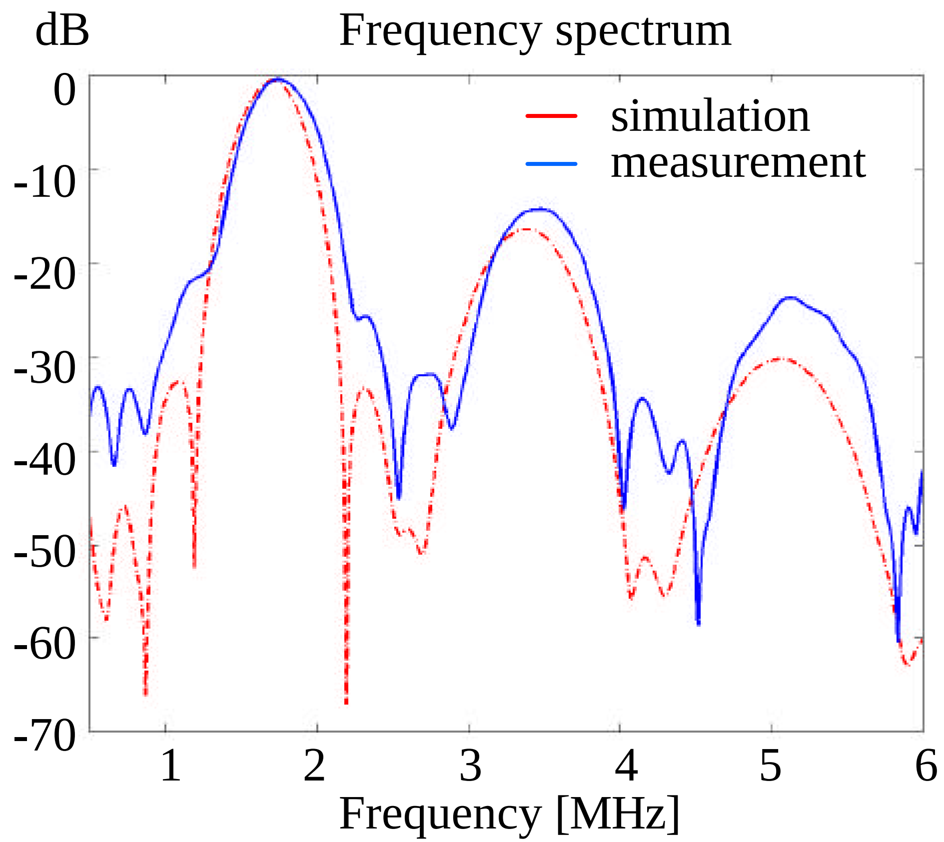
Comparison of numerical results with Fay solution at x = 5σ

High Intensity Focused Ultrasound (HIFU) Source



HIFU Source
Measurement vs. nonlinear Simulation





Piezoelectric Transducers

- Finite element formulation and simulation tasks
- Impedance and eigenfrequencies of piezoelectric cube
- Annular array antenna
- Ultrasound phased array
- SAW transducers
- Nonlinear piezoelectric material modeling
- Multilayer stack actuator

FE Modeling of Piezoelectric Transducers (I)

- Mechanical field

$$\nabla \vec{T} + \vec{f}_{\mathrm{V}} = \rho \frac{\partial^2 \vec{u}}{\partial t^2}$$

$$\vec{T} = [c]\vec{S}$$

$$\vec{S} = \mathbf{B}\vec{u}$$

$\vec{T}$	mechanical stress
$\vec{S}$	mechanical strain
$\vec{f}_{\mathrm{V}}$	volume force
$\mathbf{B}$	differential operator

- Coupling equations

$$\begin{aligned} \vec{T} &= [c]^E \ \vec{S} - [e]_t \ \vec{E} \\ \vec{D} &= [e] \ \vec{S} + [\varepsilon]^S \ \vec{E} \end{aligned}$$

$[c]^E$	mechanical material tensor
$[e]$	piezoelectric coupling tensor
$[\varepsilon]^S$	dielectric material tensor

- Electric field

$$\nabla \cdot [\varepsilon] \nabla \phi = 0$$

$\vec{D}$	electric displacement
$\vec{E}$	electric field intensity

FE Modeling of Piezoelectric Transducers (II)

- FE-formulation

$$\begin{pmatrix} \mathbf{M}_{uu} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \{\ddot{u}\} \\ \{\ddot{\Phi}\} \end{pmatrix} + \begin{pmatrix} \mathbf{C}_{uu} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \{\dot{u}\} \\ \{\dot{\Phi}\} \end{pmatrix} + \begin{pmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} \\ \mathbf{K}_{u\phi}^t & -\mathbf{K}_{\phi\phi} \end{pmatrix} \begin{pmatrix} \{u\} \\ \{\Phi\} \end{pmatrix} = \begin{pmatrix} \{F\} \\ \{Q\} \end{pmatrix}$$

$\mathbf{K}_{uu}$	mechanical stiffness matrix
$\mathbf{C}_{uu}$	mechanical damping matrix
$\mathbf{M}_{uu}$	mechanical mass matrix
$\mathbf{K}_{\phi\phi}$	dielectric stiffness matrix
$\mathbf{K}_{u\phi}$	piezoelectric coupling matrix

$\{F\}$	external mechanical forces
$\{Q\}$	electric charges
$\{u\}$	nodal vector of displacement
$\{\Phi\}$	nodal vector of scalar electric potential

Piezoelectric Transducers: Solution Algorithms

- Eigenvalue and harmonic calculations
  Standard algorithms can be easily extended or directly applied to piezoelectric systems

- Transient calculations
  Direct implicit solver (profile and sparse), not all iterative solvers applicable (e.g. CG fails due to negative diagonal elements)

- Explicit solver not applicable due to *massless* electric potential

Piezoelectric Transducers: Simulation Tasks

- Input impedance
- Transmit and receive mode
- Resonance and antiresonance calculations
  Eigenfrequency calculations with different electrical boundary conditions
- Determination of electro-mechanical coupling coefficient
  Coupling coefficient *k* defined as

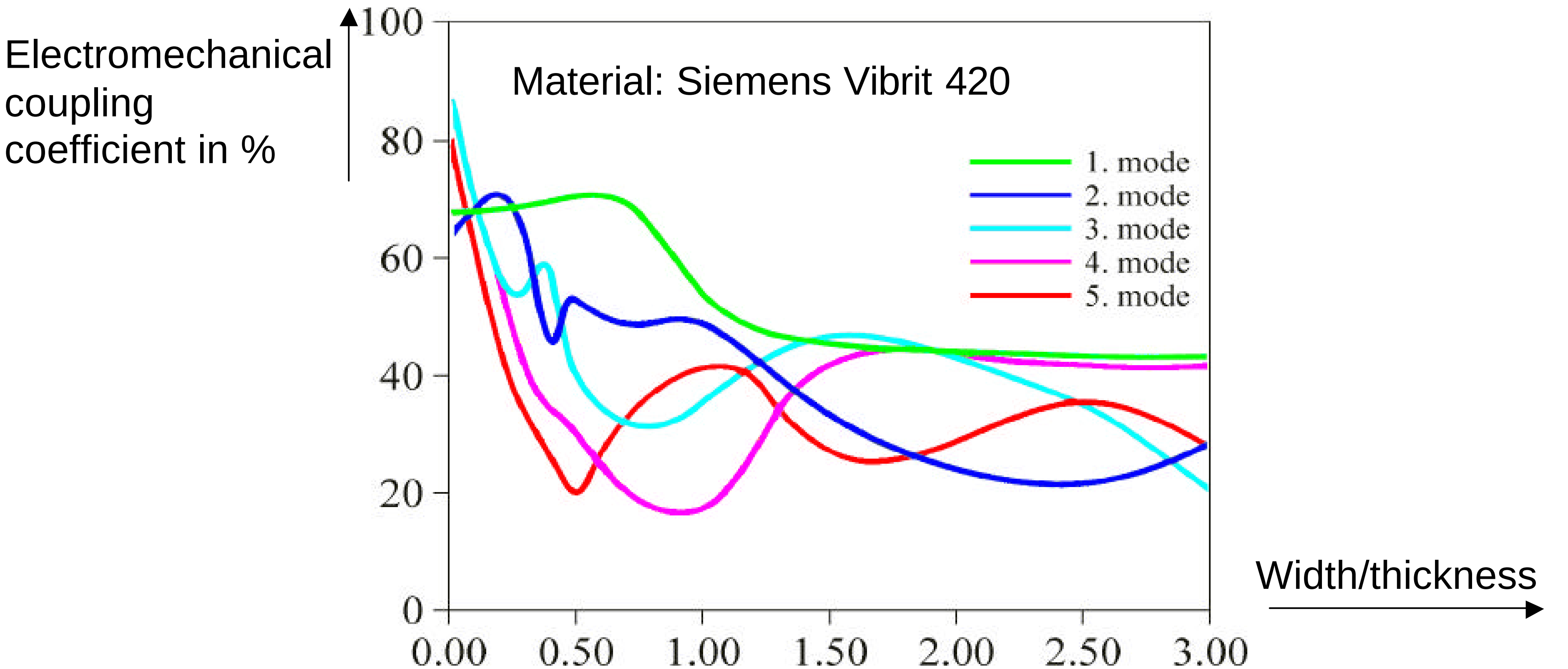
$$k^2 = E_k^2/(E_m \cdot E_d)$$

  with E_k, E_m, and E_d the coupling, mechanical and dielectric energy
  Typically, *k* is calculated by means of

$$k^2 = \left(w_a^2 - w_r^2\right)/w_a^2$$

  (Requires both resonance and antiresonance calculations)

Electromechanical Coupling for Long Piezoelectric Bars



Calculation of the Input Impedance of Piezoelectric Transducers

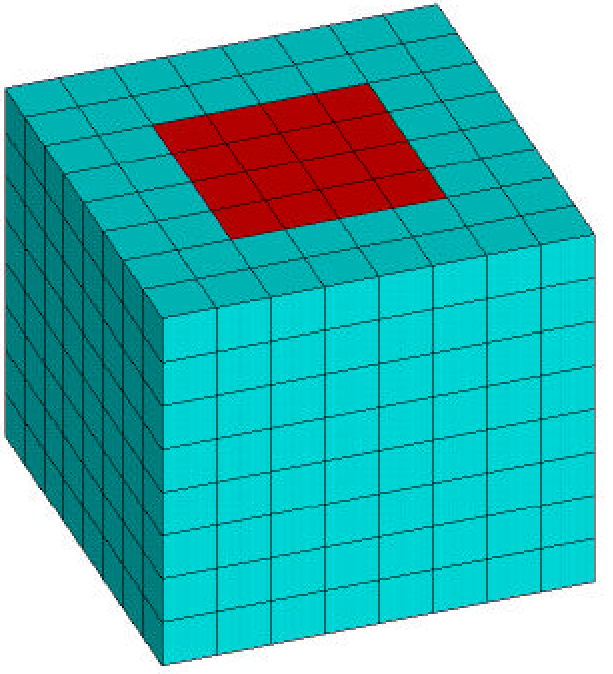
Several approaches available

- Calculation in frequency domain at discrete frequencies
  Uneffective, since a large number of frequencies may be required
- Calculation in time domain
  A short electric pulse is applied to the transducer and the electric input impedance is calculated by means of the Fourier Transform
- Potential excitation
  Requires calculation of the surface charge on the electrodes
- Charge excitation
  Impedance is calculated as

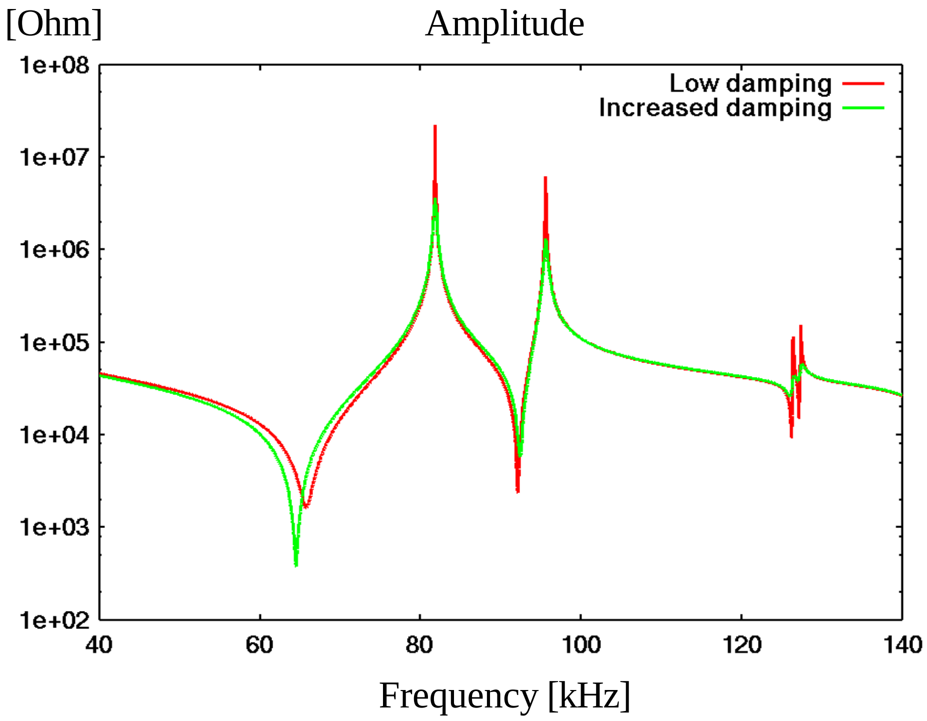
$$Z(\omega) = -\frac{j\phi(\omega)}{\omega Q(\omega)}$$

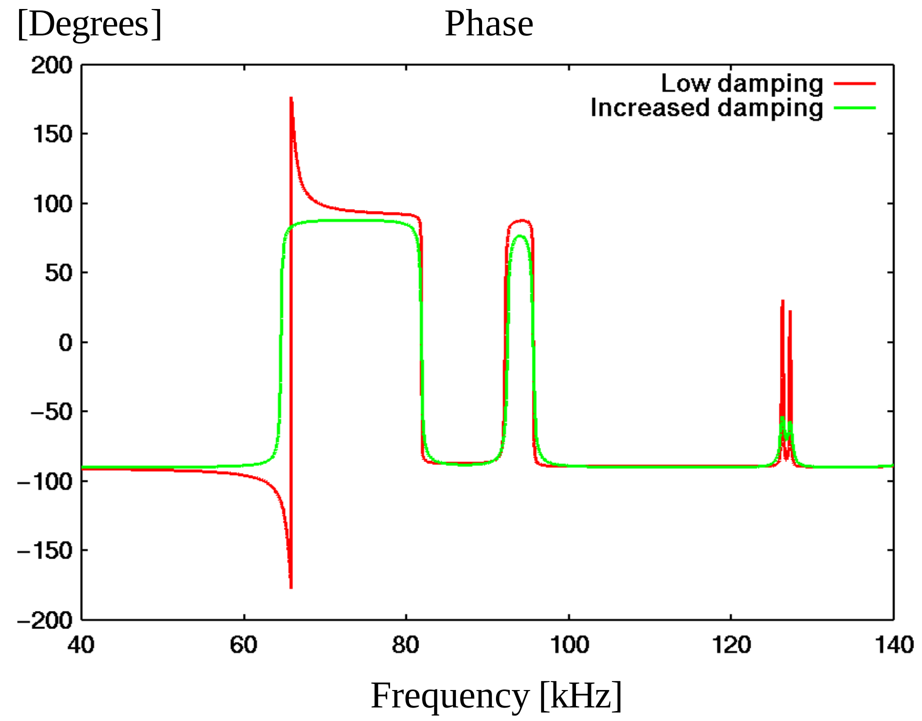
Input Impedance of PZT-4 Cube

- Piezoelectric cube of PZT-4 of sidelength 2 cm
- Only partially electroded top and bottom surfaces
- 3 symmetry planes
  ⇒ Use only 1/8-th of cube in simulation
- Symmetry boundary conditions
  no x-displacement on yz-symmetry plane
  no y-displacement on xz-symmetry plane
  no z-displacement and grounded electric potential on xy-symmetry plane
- Electric boundary conditions
  Top electrode realized as an equipotential area
  Short charge pulse (Dirac-like) is applied to the top electrode of the transducer



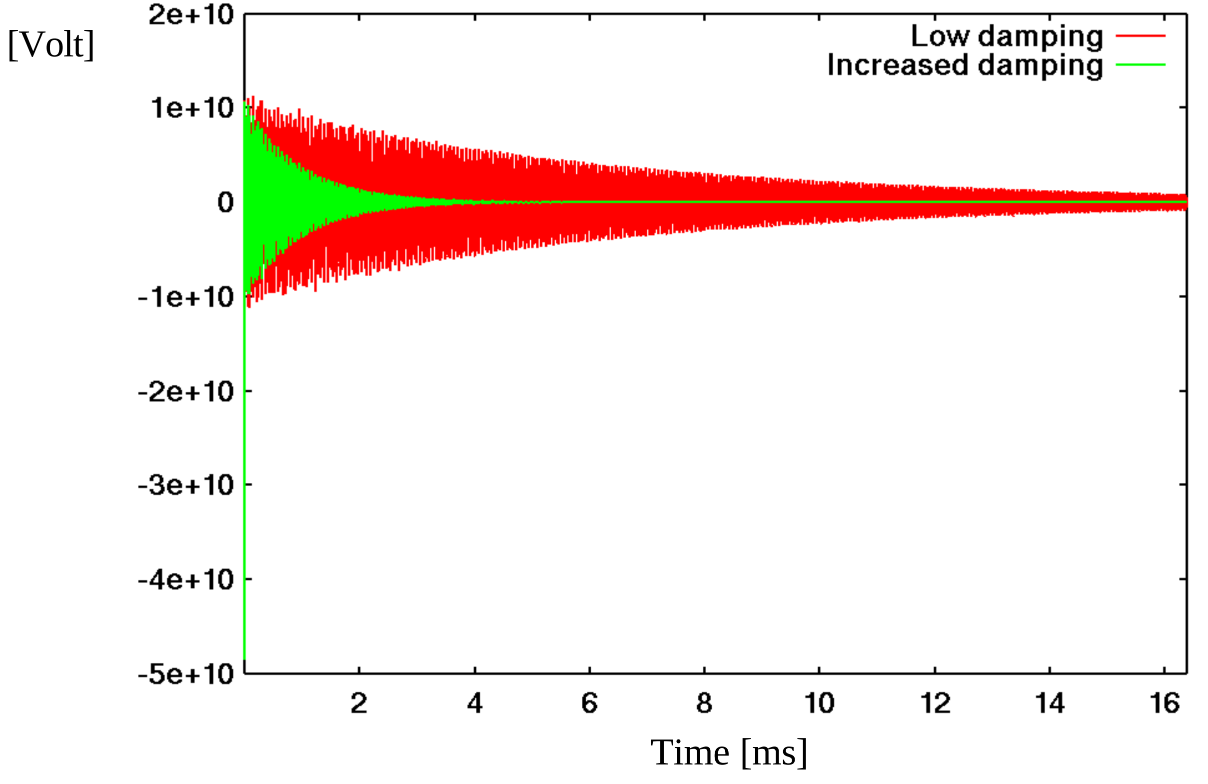
Calculated Input Impedance of PZT-4 Cube





- Only material damping has been modified between simlations
- All other simulation data identical

PZT-4 Cube: Electric Potential on Top Electrode



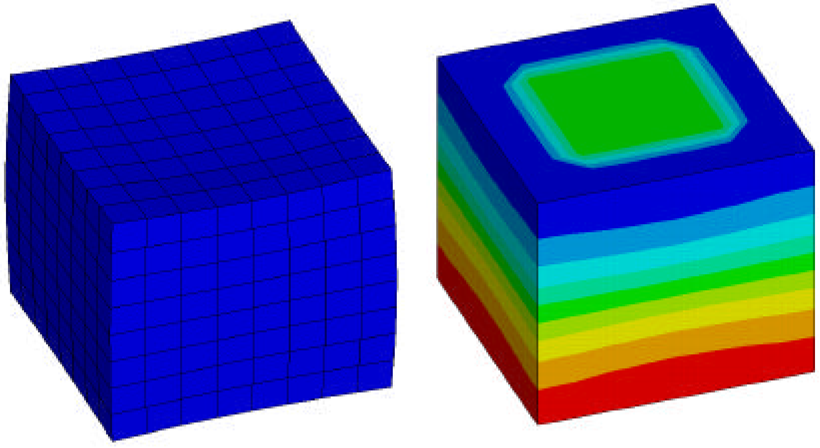
- Low damping results in longer time signal
- Cut-off error in Fourier analysis results in phase-errors of impedance

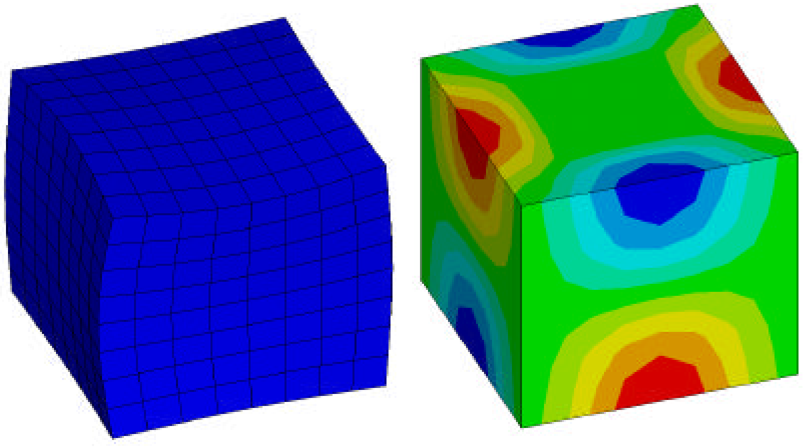
Eigenfrequency Calculations of PZT-4 Cube

- Same geometry as in impedance calculations
- Resonance and antiresonance calculations show 8 mode pairs below 140 kHz
- Only 4 mode pairs shown in impedance calculations (indicated by arrows)
- Zero-coupling modes
- Must be accounted for in coupling factor calculations

Antiresonance frequencies [kHz]	Resonance frequencies [kHz]
70.2591	68.8002
72.4046	70.2591
82.2361	72.4046
92.3716	92.3716
96.2762	93.4206
122.543	122.543
128.050	127.970
129.021	128.980

Eigenmodes of PZT-4 Cube



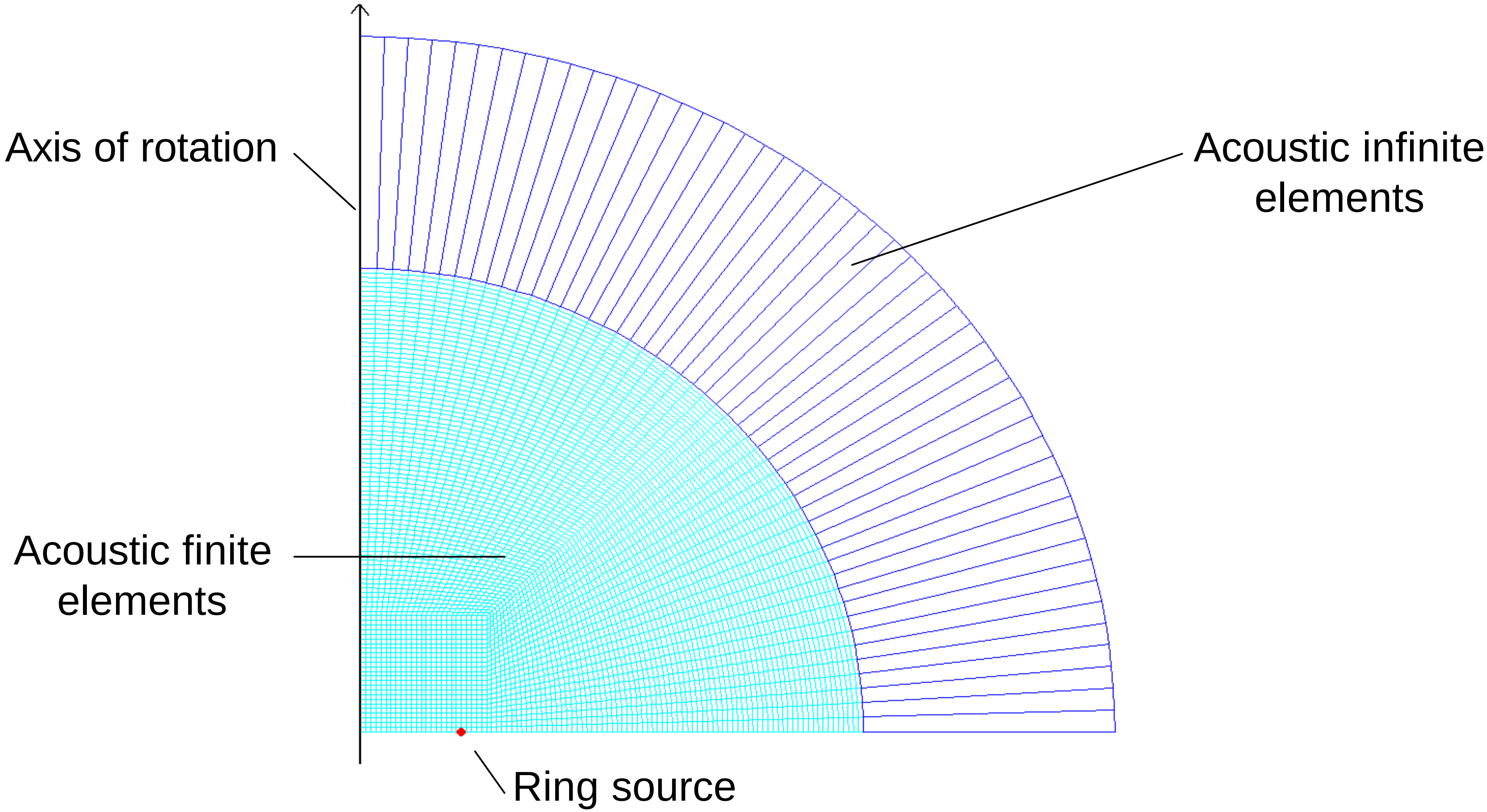


Mode shape and electric potential distribution of 1st resonance mode (coupling)

Mode shape and electric potential distribution of 2nd resonance mode (non-coupling)

Piezoelectric Annular Array Antenna

Finite Element Mesh of Acoustic Ring Source



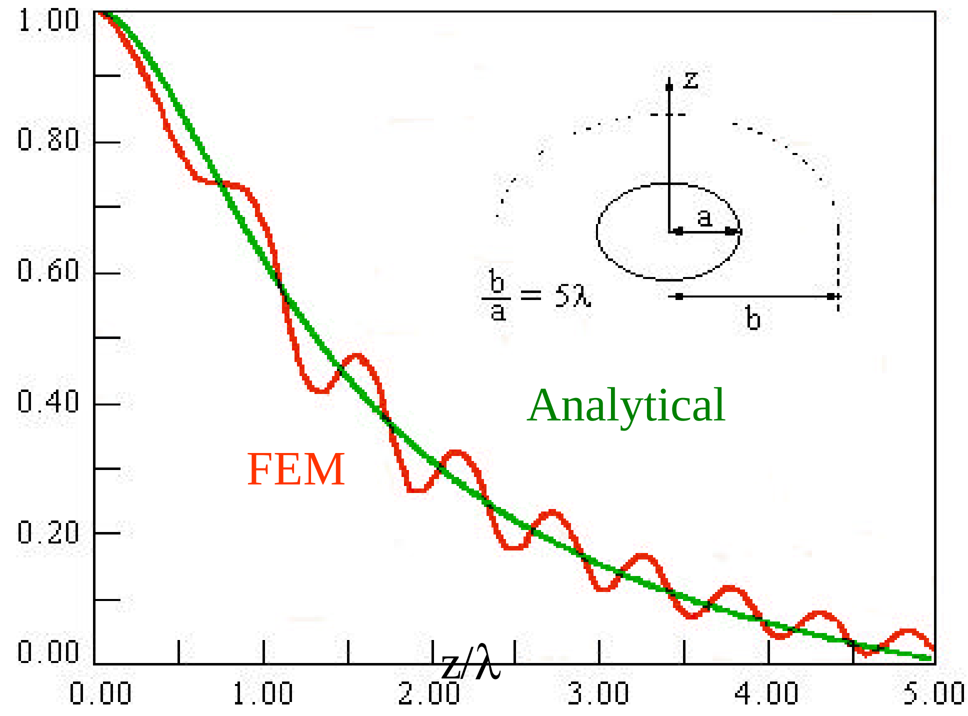
Finite Element Simulation of Acoustic Ring Source

Problems

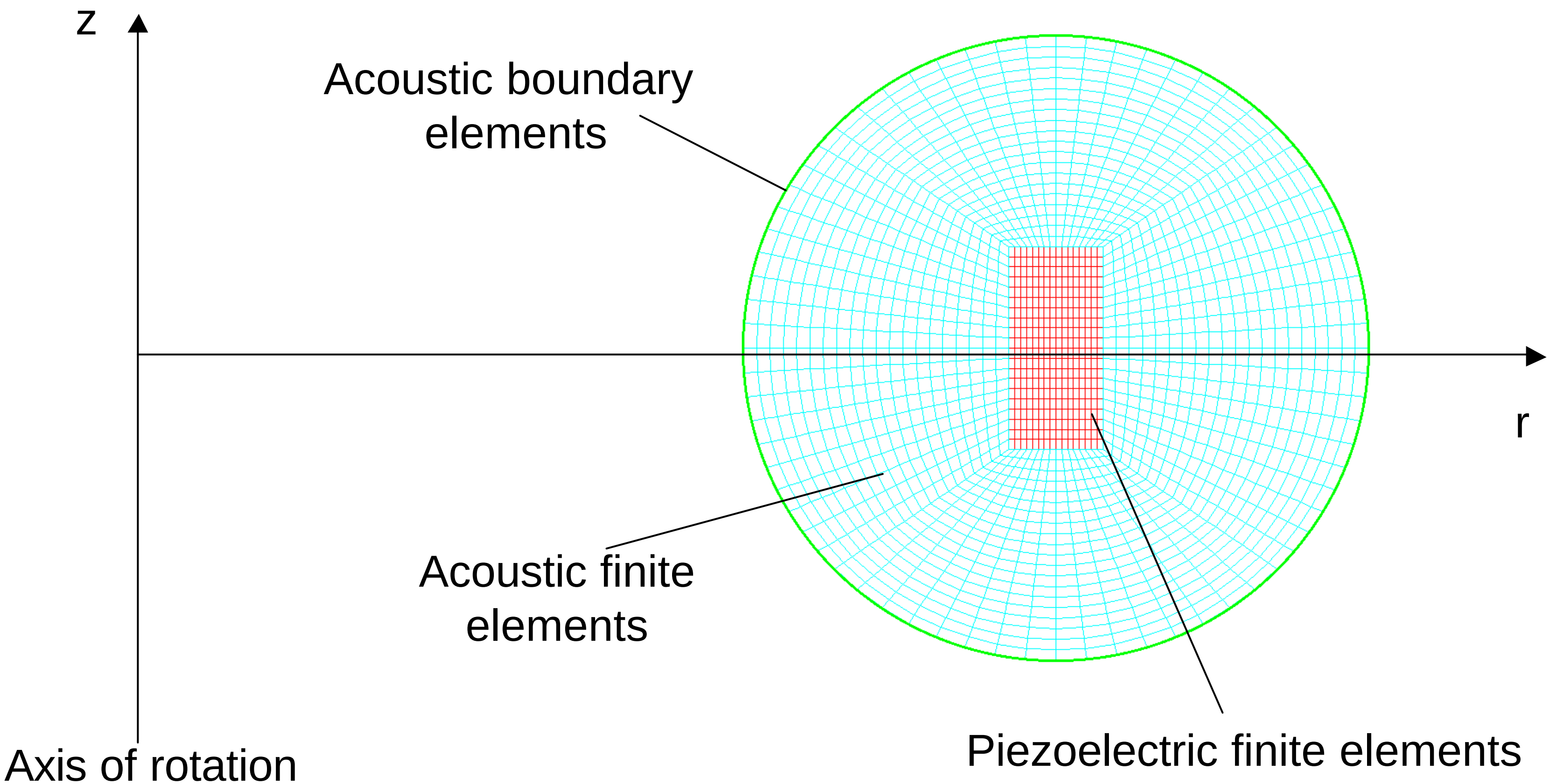
- Standing waves due to reflections on the boundary
- Infinite elements must be located in the far field
- Near field length strongly depends on the the diameter of the source and may become extremely large

Solution

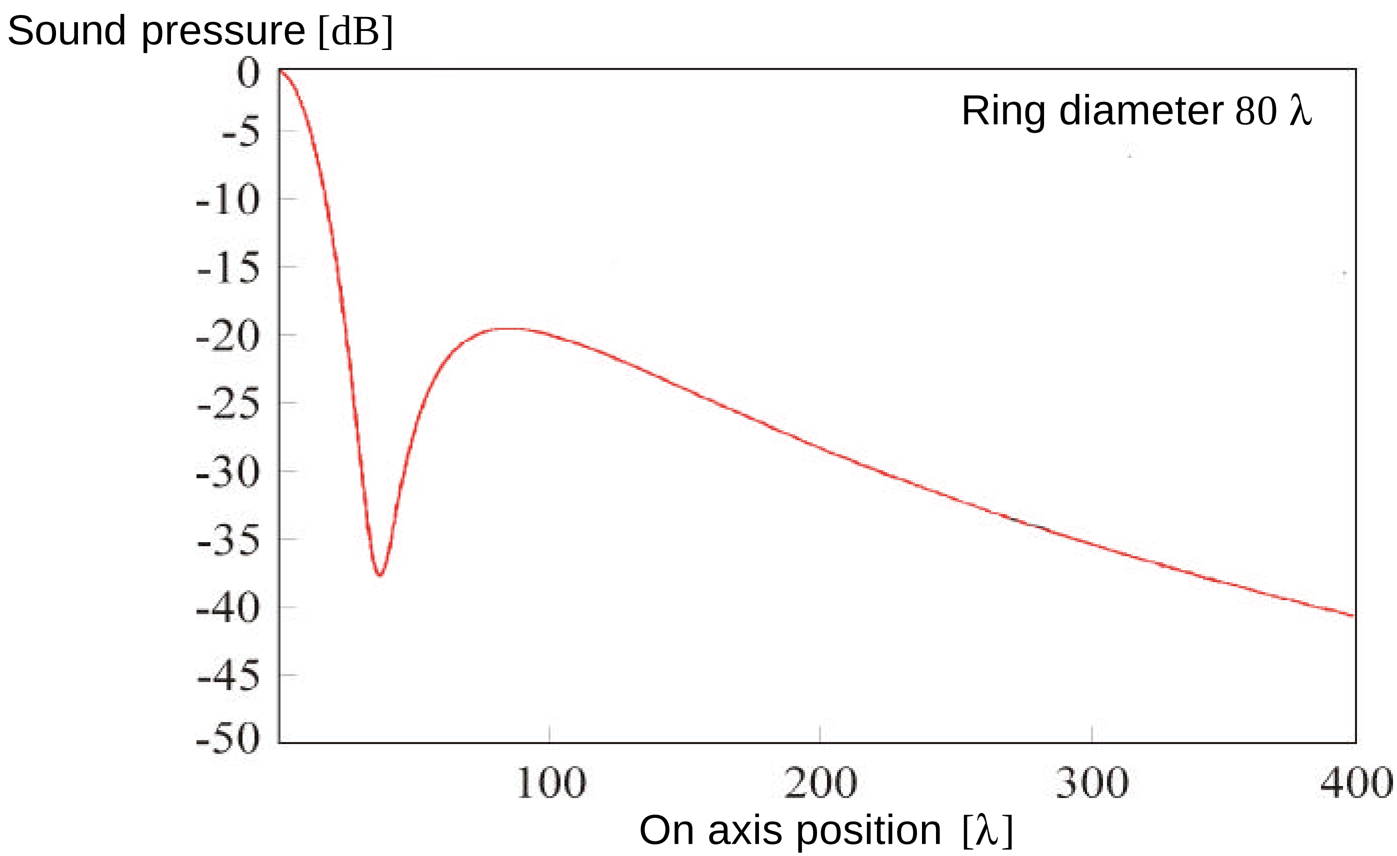
- Coupled FEM-BEM approach



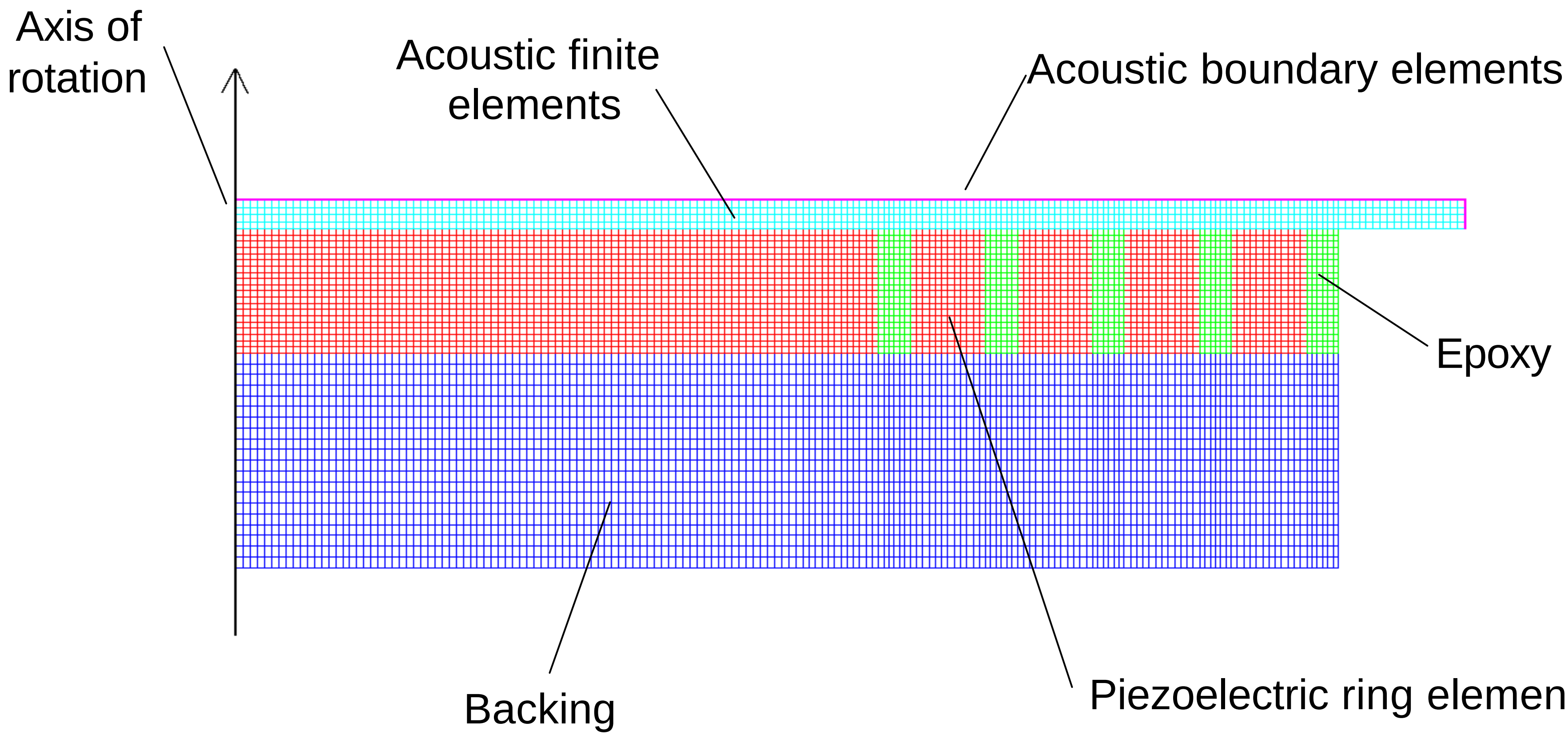
FEM-BEM Model of a Piezoelectric Ring Transducer



On-axis Pressure of Piezoelectric Ring Transducer



FEM-BEM Model of an Annular Array



FEM-BEM Modeling of an Annular Array Antenna

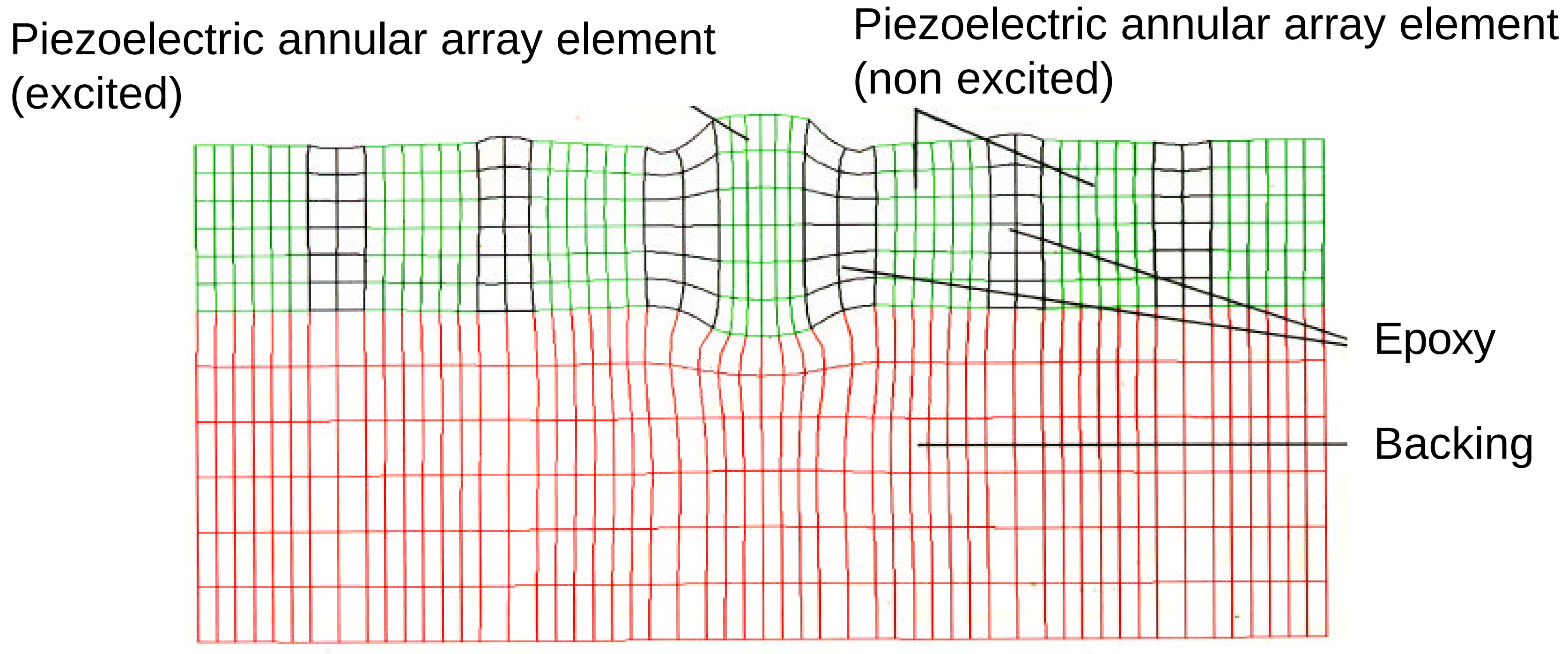
Modeling approach

- Boundary elements efficiently applied only to harmonic problems
- Transient problems can not be treated directly
- Use Fast Fourier Transform, to split transient problem into separate harmonic problems
- Use inverse Fourier Transform to combine the results of the harmonic problems and get final transient solution

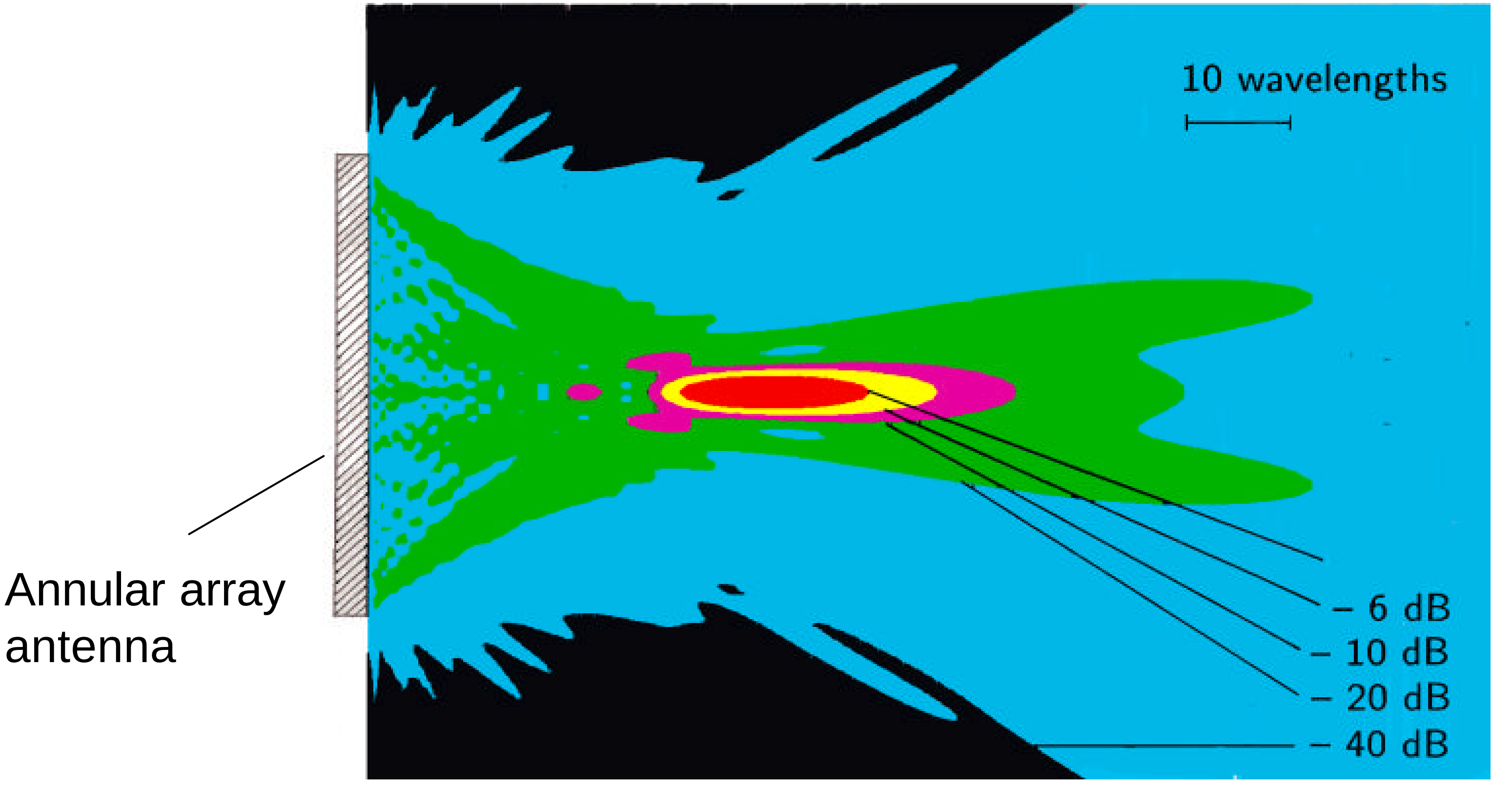
Limitations

- Applicable only to cases, in which a small number of single frequency runs is sufficient
- Sensible to phase errors in inverse Fourier Transform

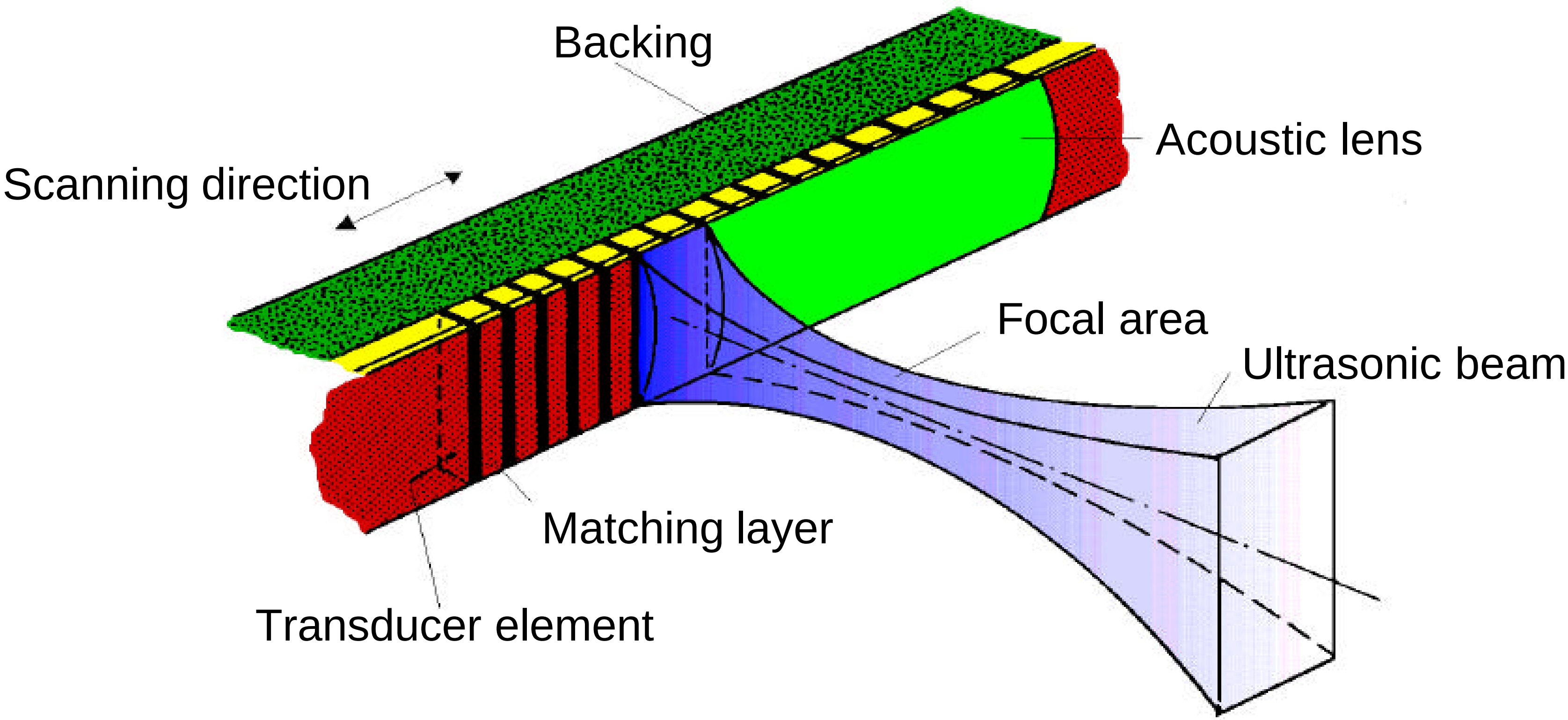
Mechanical Deformation of an Annular Array Antenna



Pressure Field (isobars) of Annular Array Antenna as Computed with FEM-BEM Method



Principle of Ultrasonic Phased Array Antenna

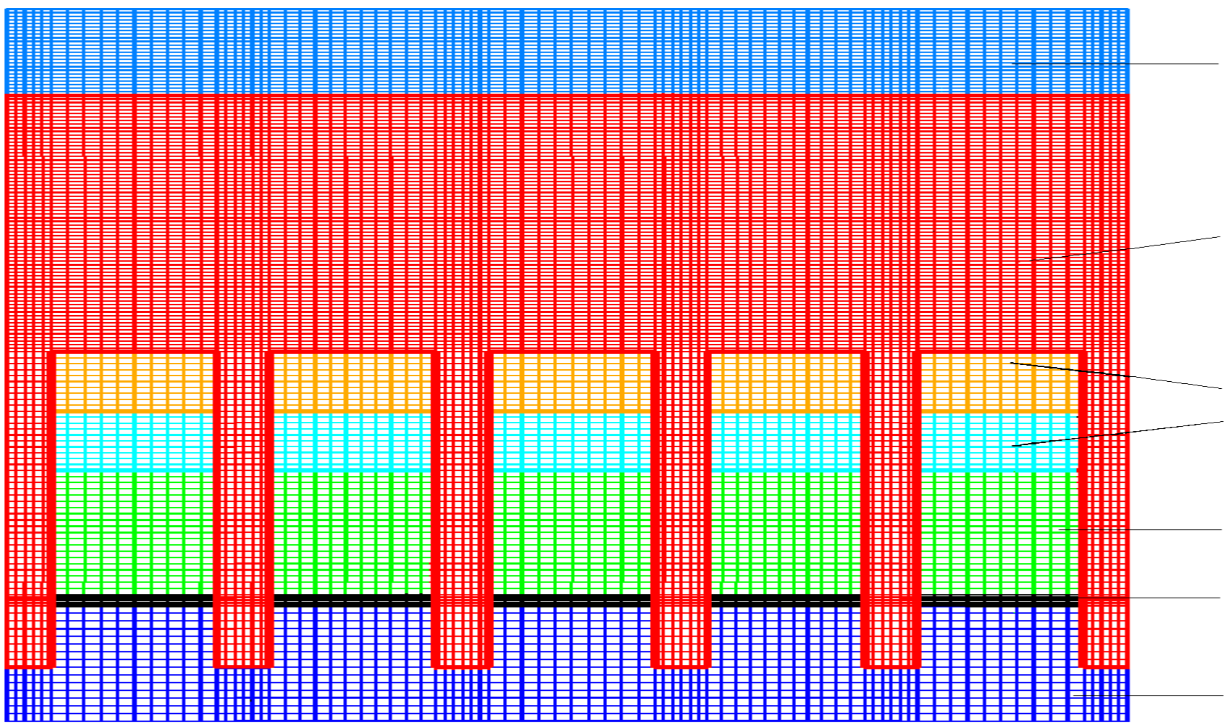


Finite Element Simulation of Phased Array Antennas

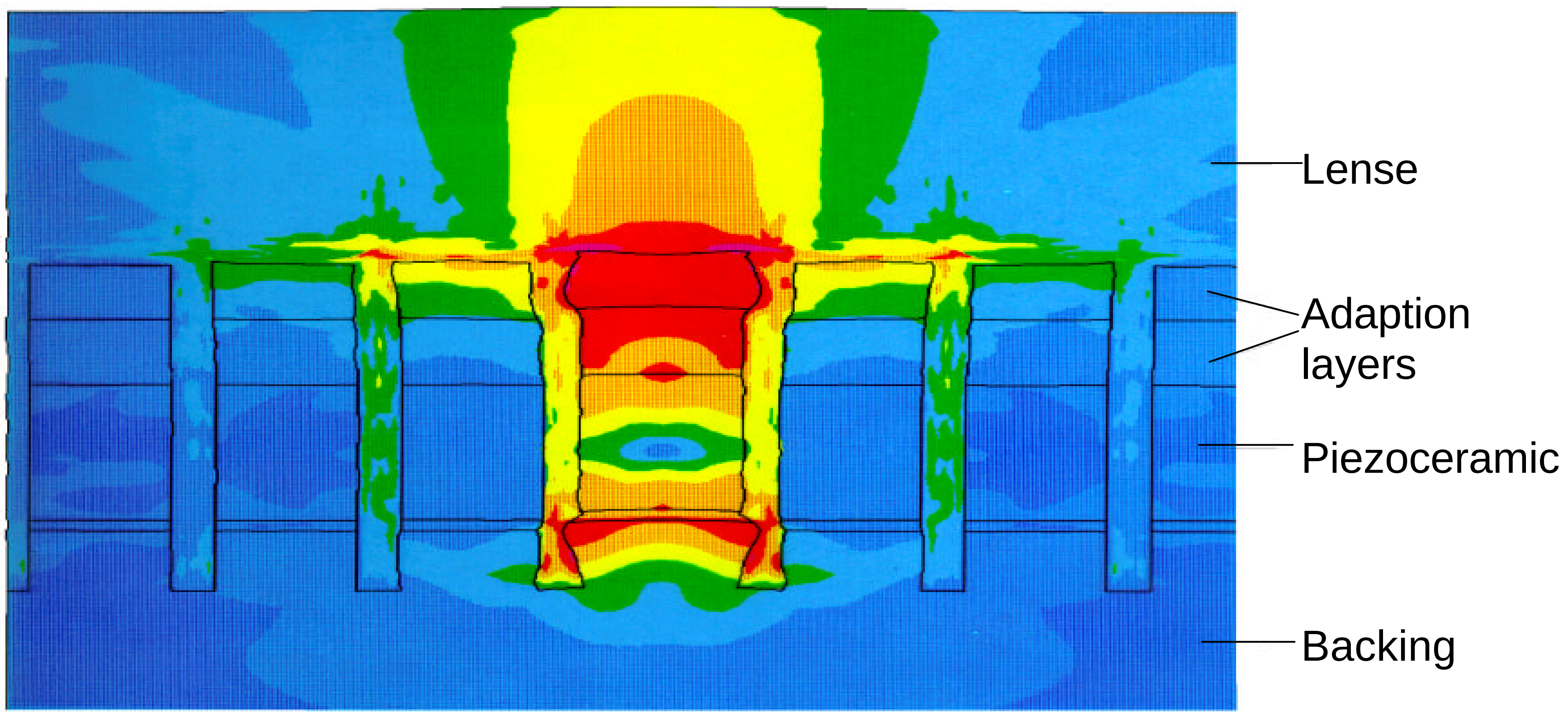
Standard simulation tasks

- Crosstalk
  Study influence of saw-cut fillings, subdicing, etc.
- Pressure pulse signals and radiation patterns
  Optimize pulse duration and directivity
- Electrical input impedance
- Pulse-echo behavior
  Most complex simulation
  Requires coupling to external electrical network

2D Finite Element Mesh of Phased Array Antenna

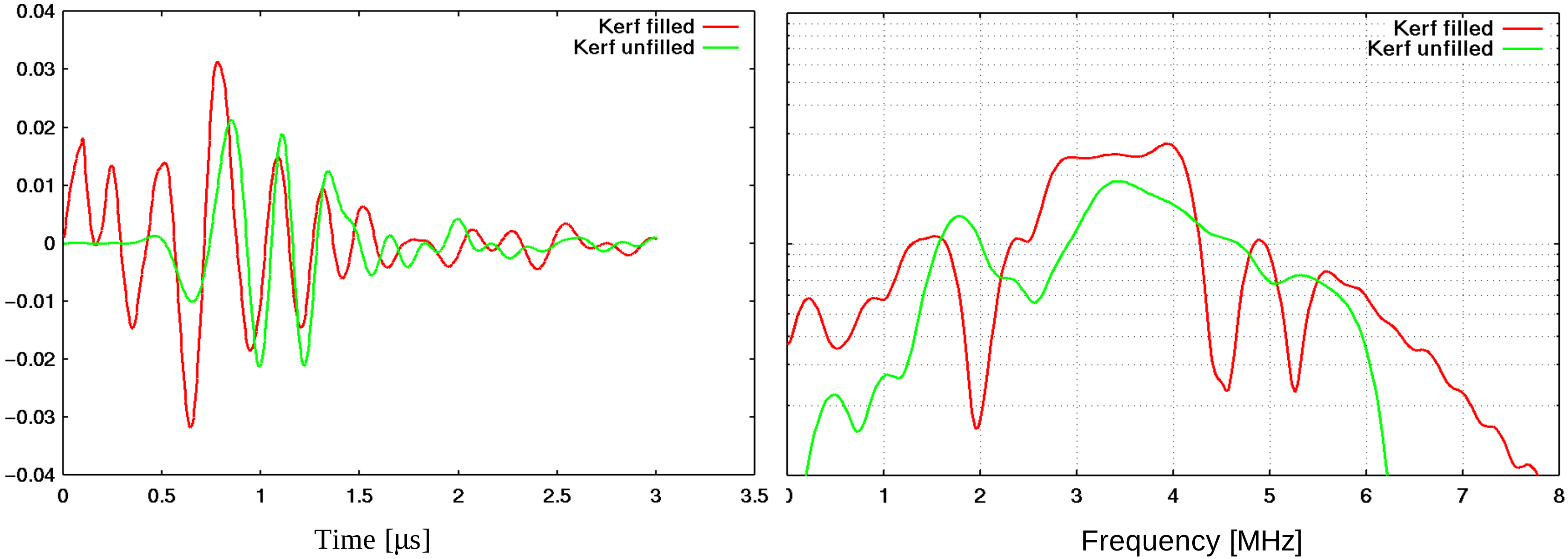


Mechanical Displacements in a Phased Array Antenna



Simulation of Crosstalk in a Phased Array Antenna

- Center transducer element excited by potential, charge, or pressure pulse
- Study electric potential and average displacements on neighbor elements



Electric crosstalk signal at first neighbor element

Calculation of Radiated Pressure

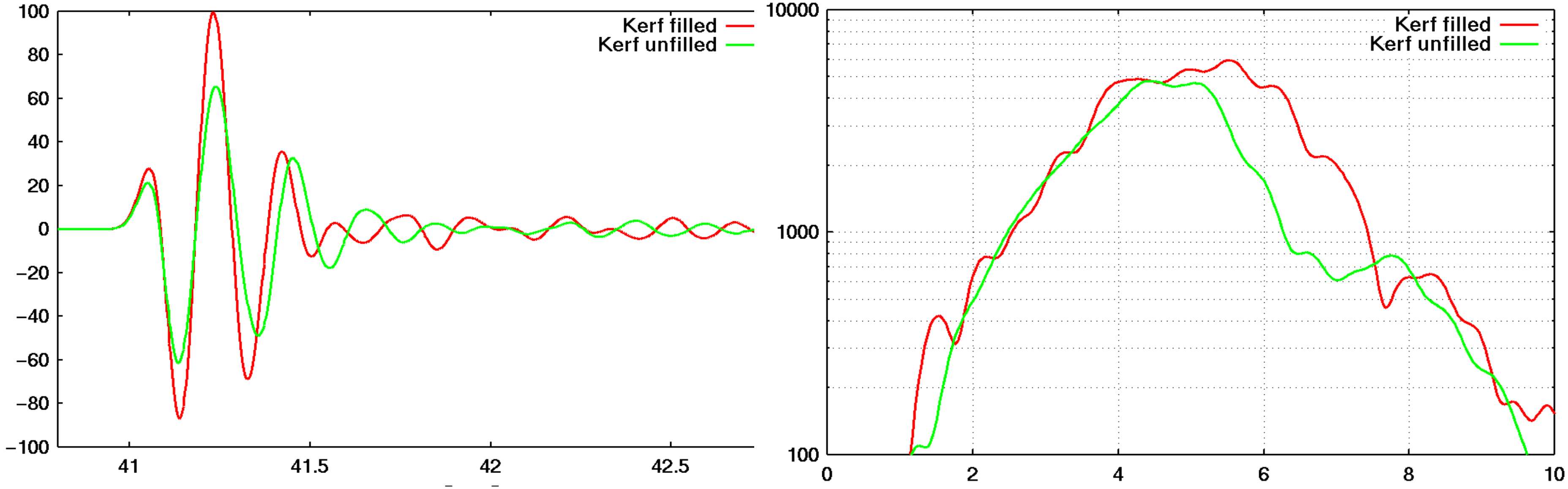
- Array antenna excited by short potential pulse
- Small distances:
  calculate pressure by pure finite element calculation
- Large distances:
  use results from FEM simulation on top of lense and
  Huygens/Kirchoff integral representation

$$p(\vec{x},t) = \frac{1}{4\pi}\int_\Gamma \left[\frac{\rho}{r}\frac{\partial v_n(\tau)}{\partial t} - \frac{\partial r}{\partial n}\left(\frac{p(\tau)}{r^2} - \frac{1}{rc}\frac{\partial p(\tau)}{\partial t}\right)\right] d\Gamma$$

$$\tau = t - r/c$$

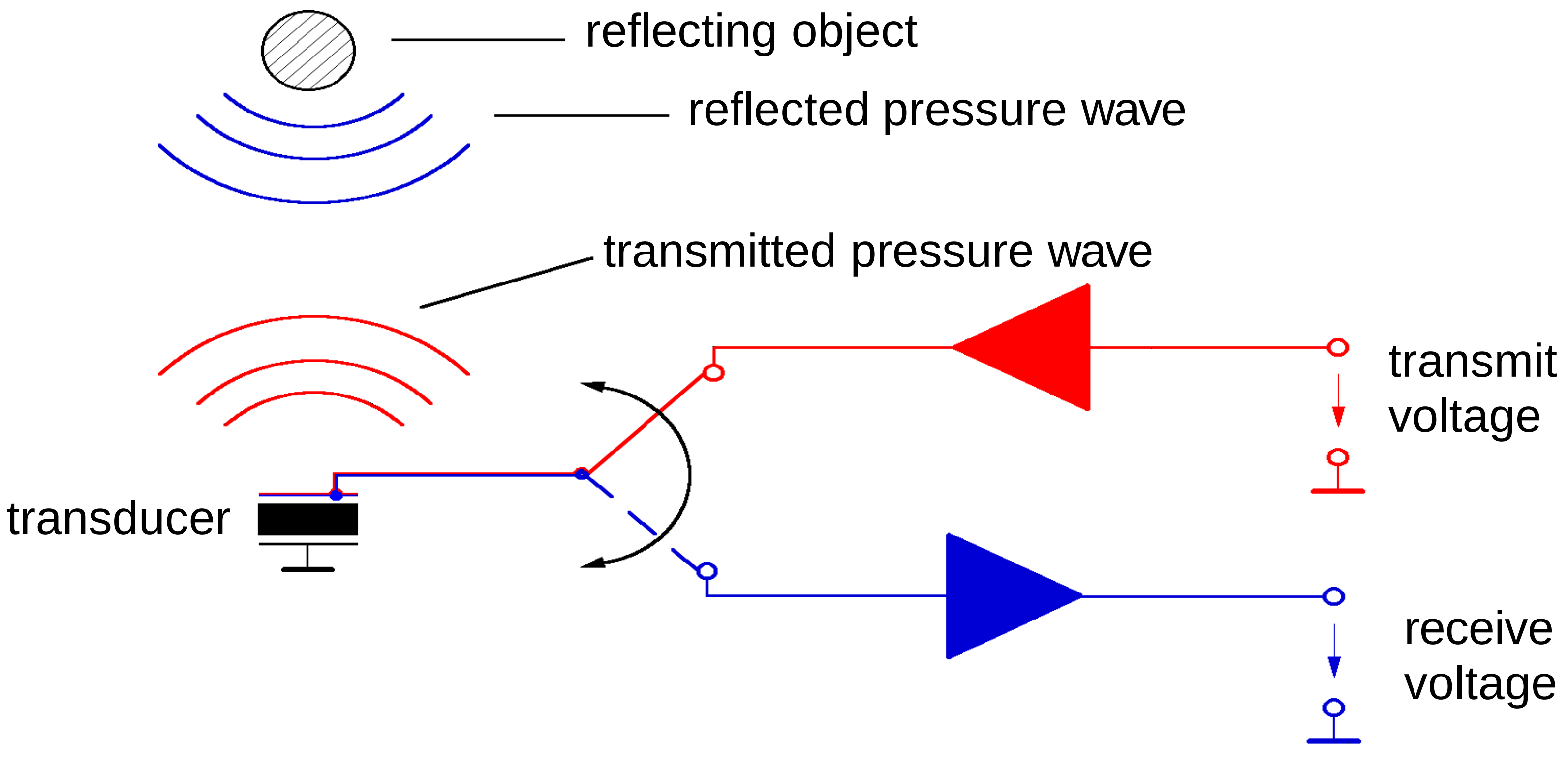
- *r* denotes the distance of a location on the surface G and the point x and t the retarded time given by

Simulation of Radiated Pressure



Calculated pressure signals and corresponding spectra
(On-axis, distance from array 60 mm)

Principle of Pulse-Echo Mode of Transducers



Simulation of Pulse-Echo Mode of Transducers

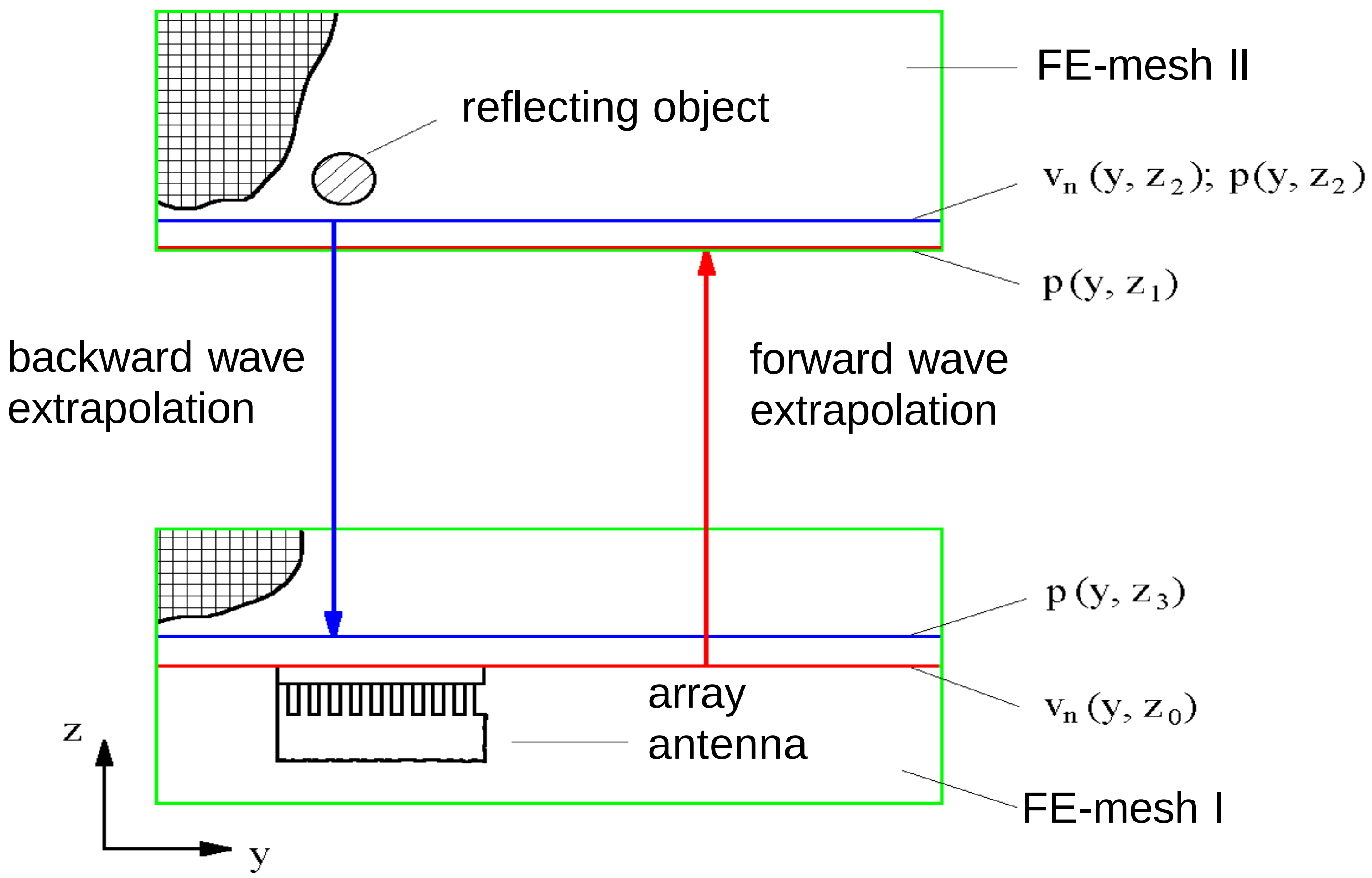
Problems encoutered in Simulation

- Requires switching from transmit to receive mode
- Reflector distance typically 50-100 wavelengths
  Straight forward approach requires tremendous finite element mesh and number of time steps
- Long calculation times

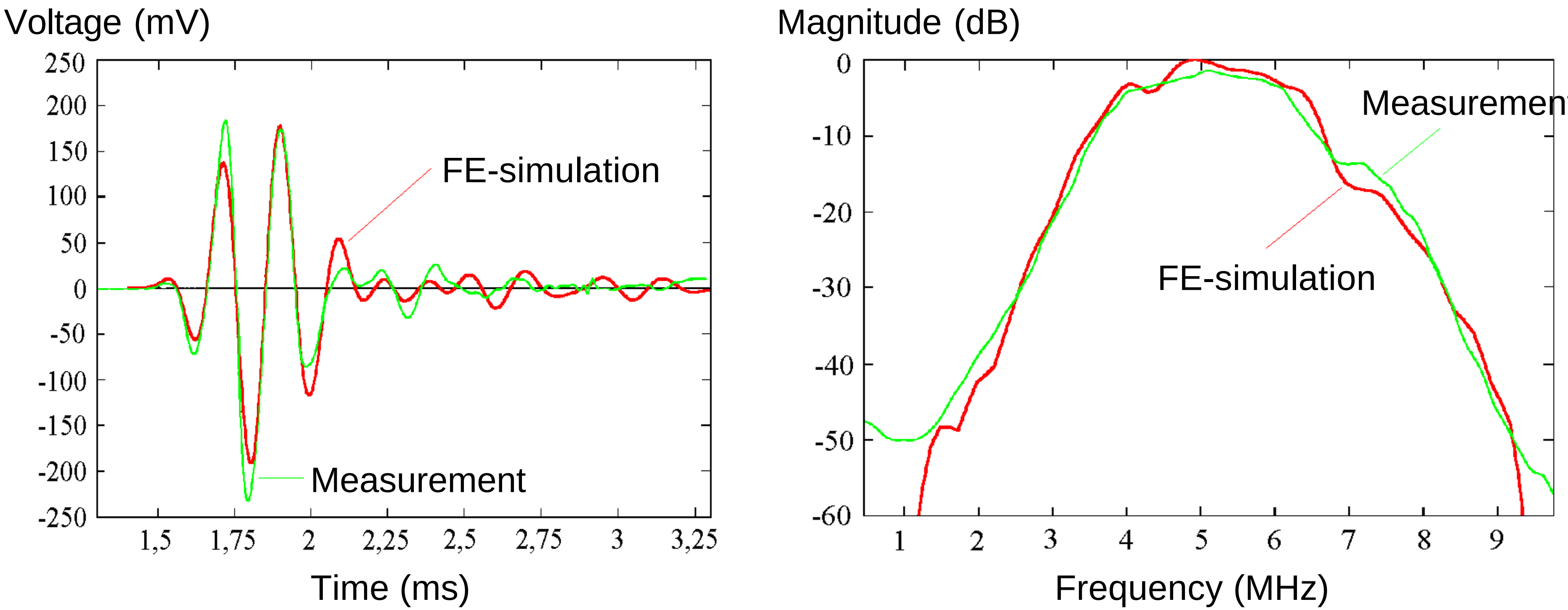
Solution

- Hybrid simulation based on 3 finite element calculations and
  forward and backward wave extrapolation by Huygens-Kirchhoff integrals
- Hide complexity by means of dedicated user interface

Simulation of Pulse-Echo Mode

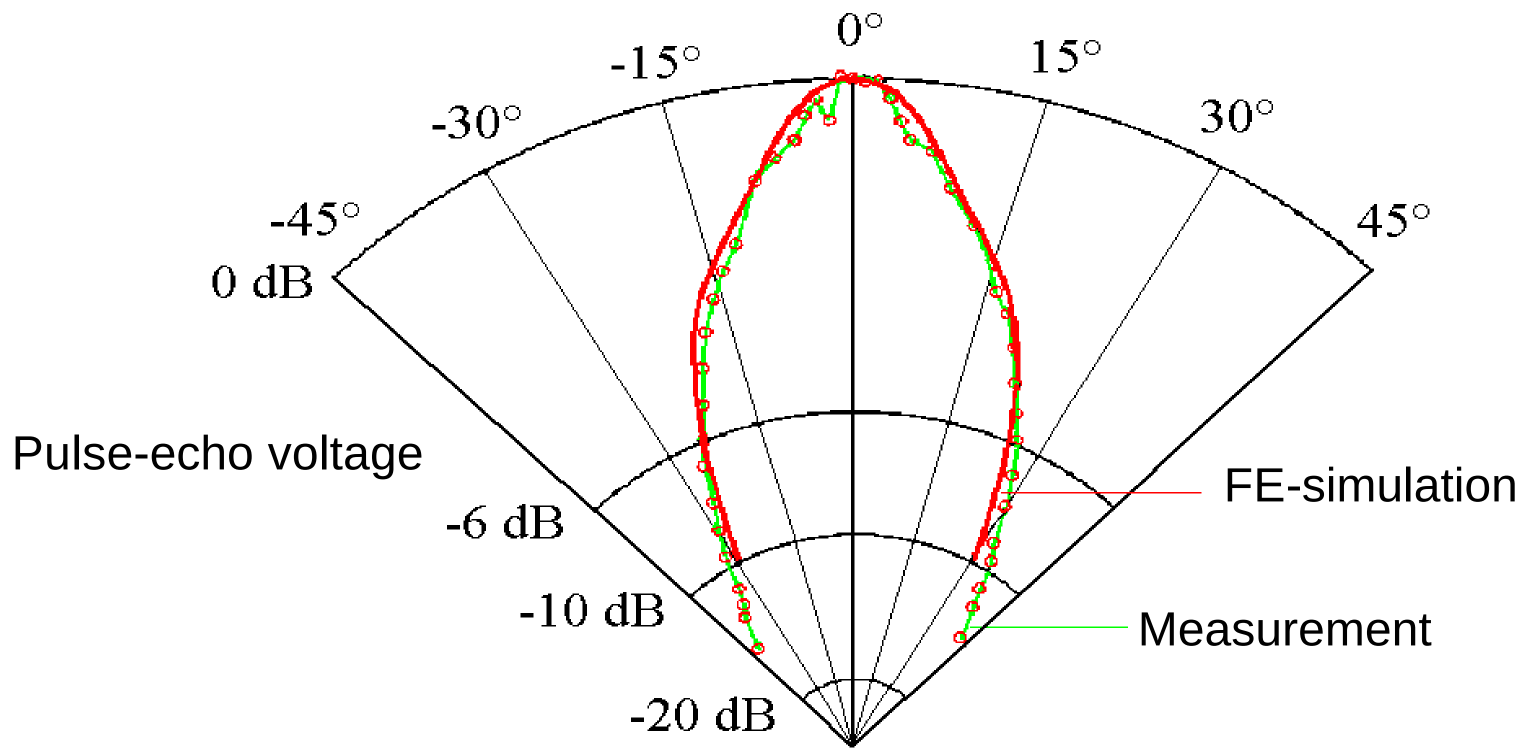


Pulse-Echo Simulation of Phased Array Antenna



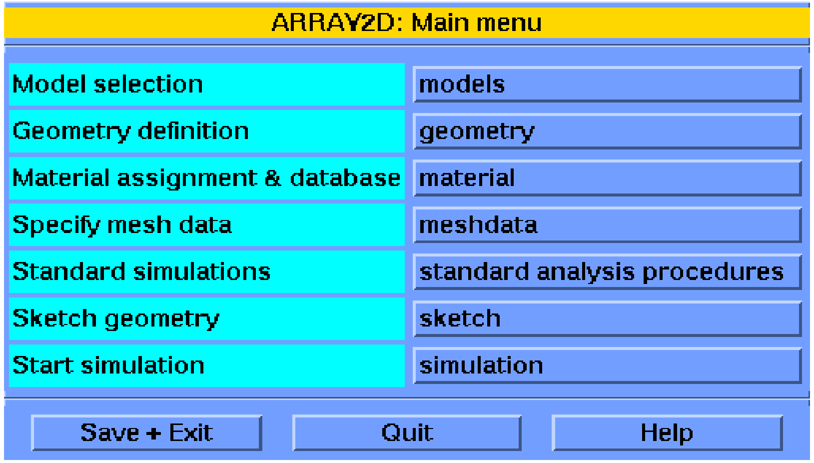
Comparison of measured and computed pulse echo signal and spectrum

Pulse-Echo Directivity Pattern of Phased Array Antennas

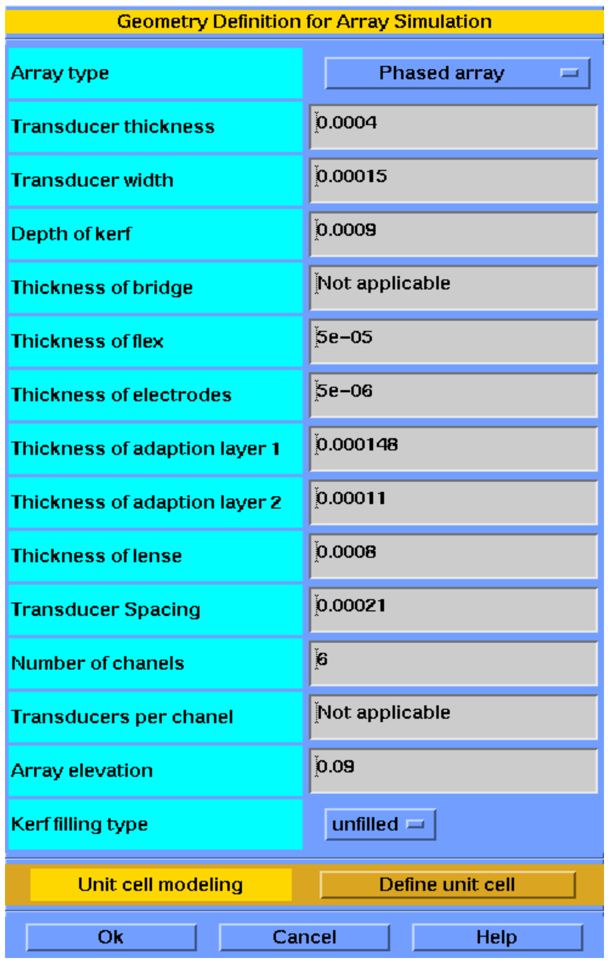


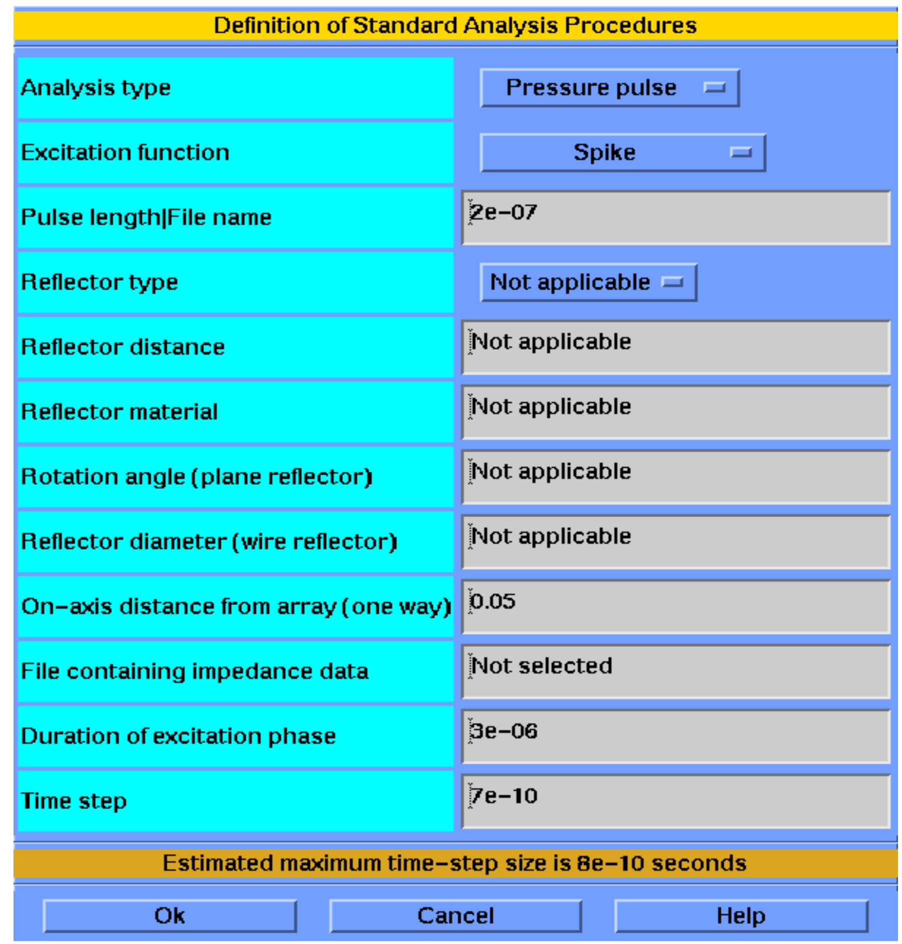
Simulation Tool for Modeling of Phased Array Antennas

- Dedicated user-interface for a specific type of applications
- Hide complexity and minimize user interaction
- Standard arrays and flexibility by unit-cell for non standard arrays
- Standard simulation tasks
- Physically mesh density definition
- Material database
- Graphic control of material damping
- All data saved in model files

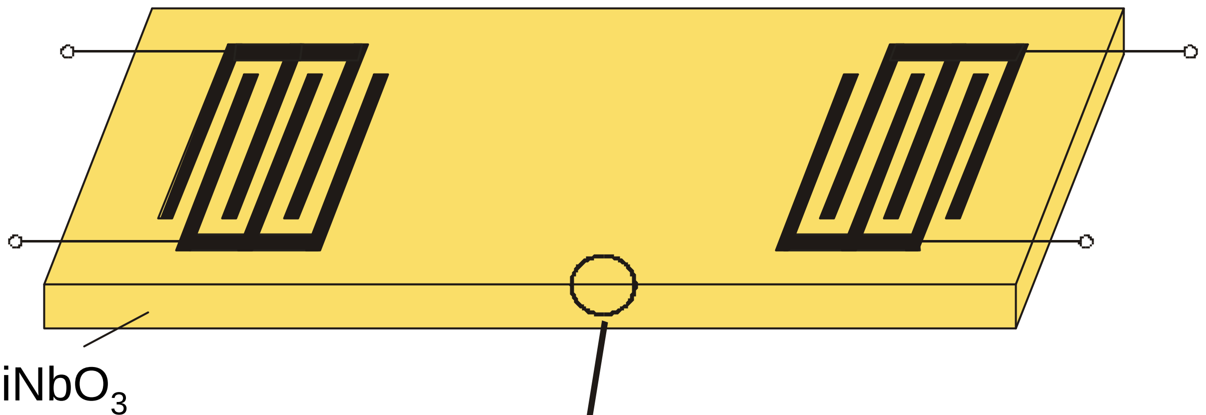


Simulation Tool for Modeling of Phased Array Antennas

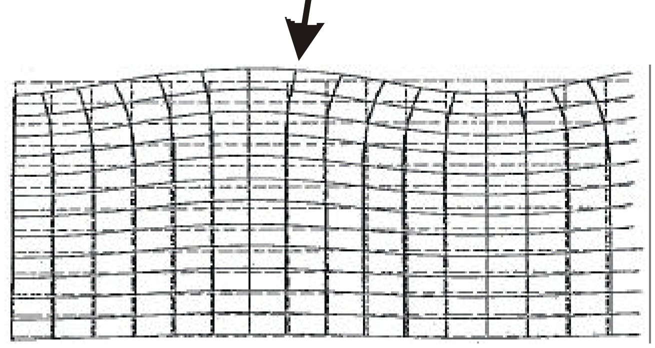




Surface Acoustic Wave Transducer

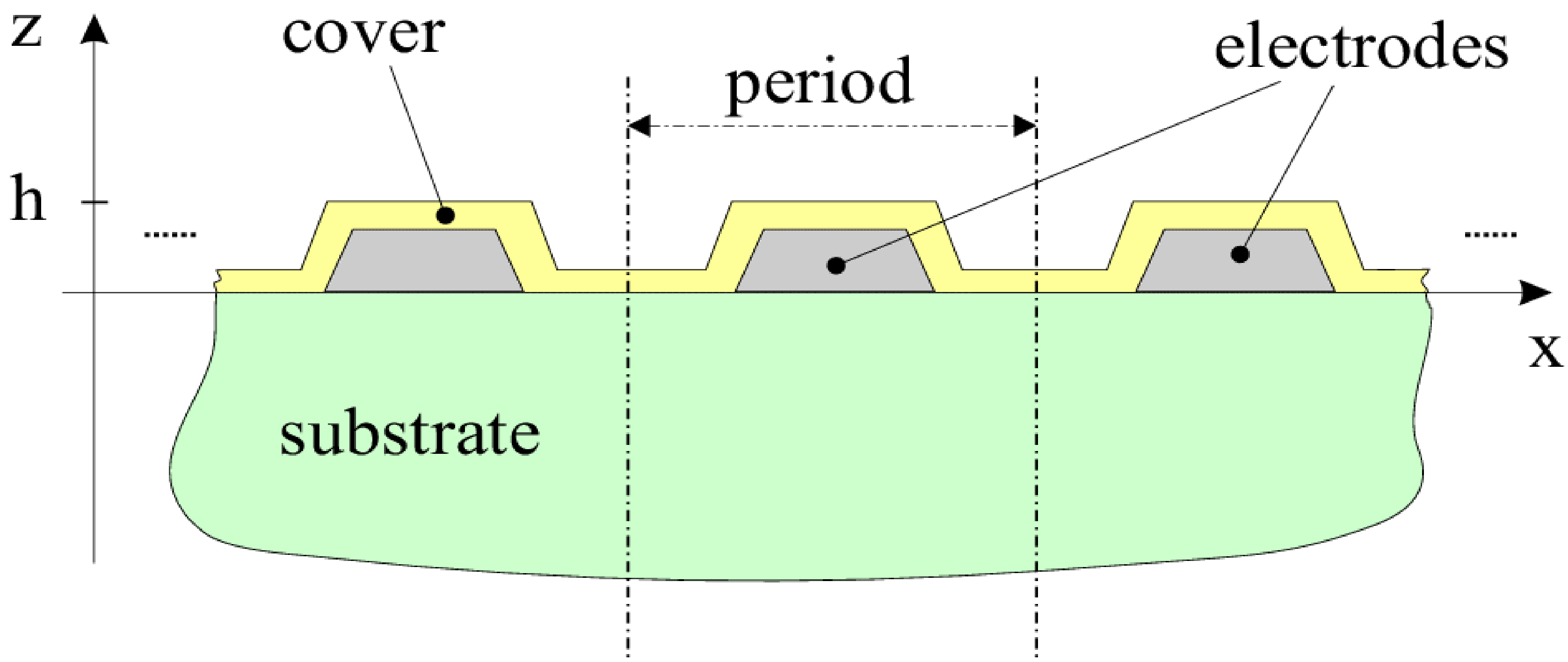


Mechanical Displacements



Center Frequency:
30 MHz – 3 GHz

2D SAW Model



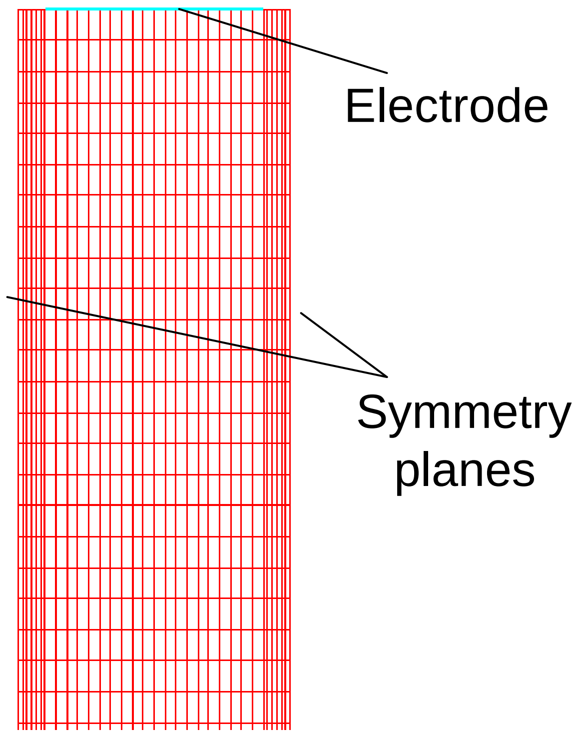
Eigenfrequency calculations of a SAW Transducer

Modeling approach

- A l/2-section of the transducer is sufficient
- Depth of the model must be chosen large enough, so that cut-off condition does not influence results

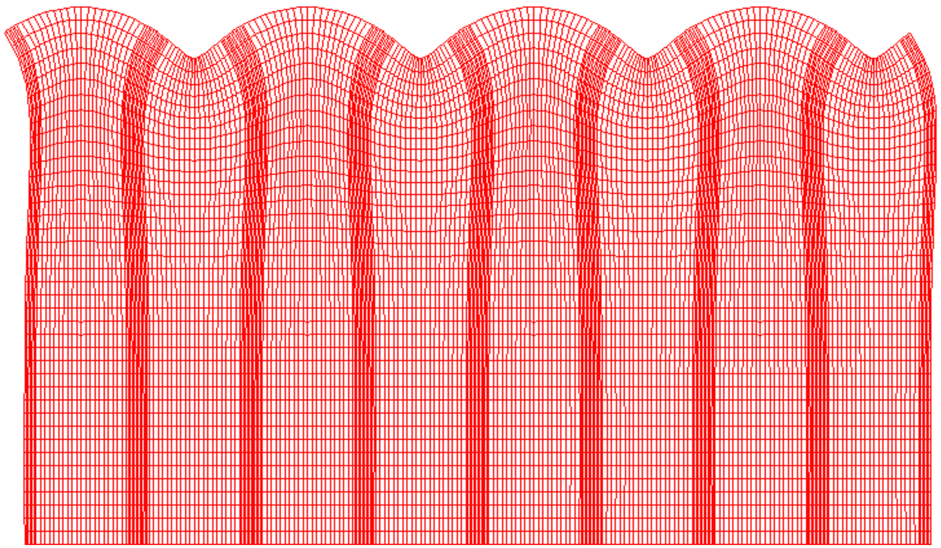
Boundary conditions

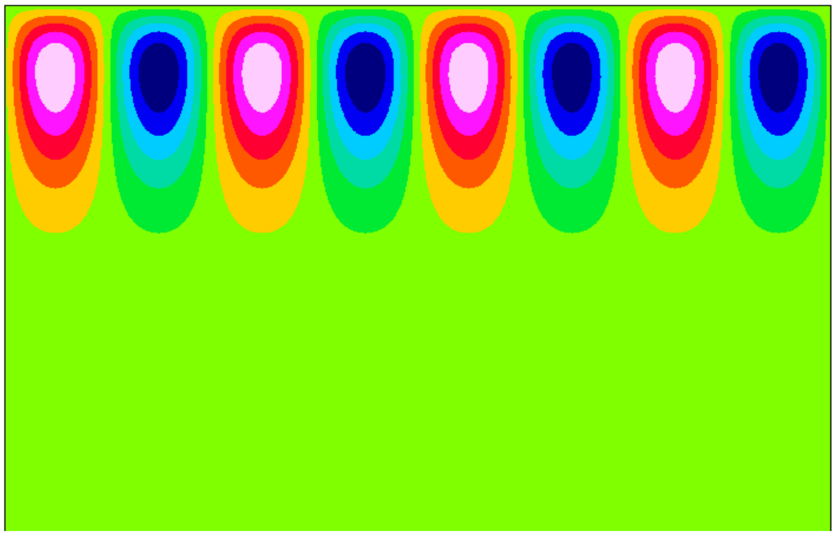
- Top electrode grounded
- Constrain left and right sides of the model to account for l/2-condition



Finite element model (detail)

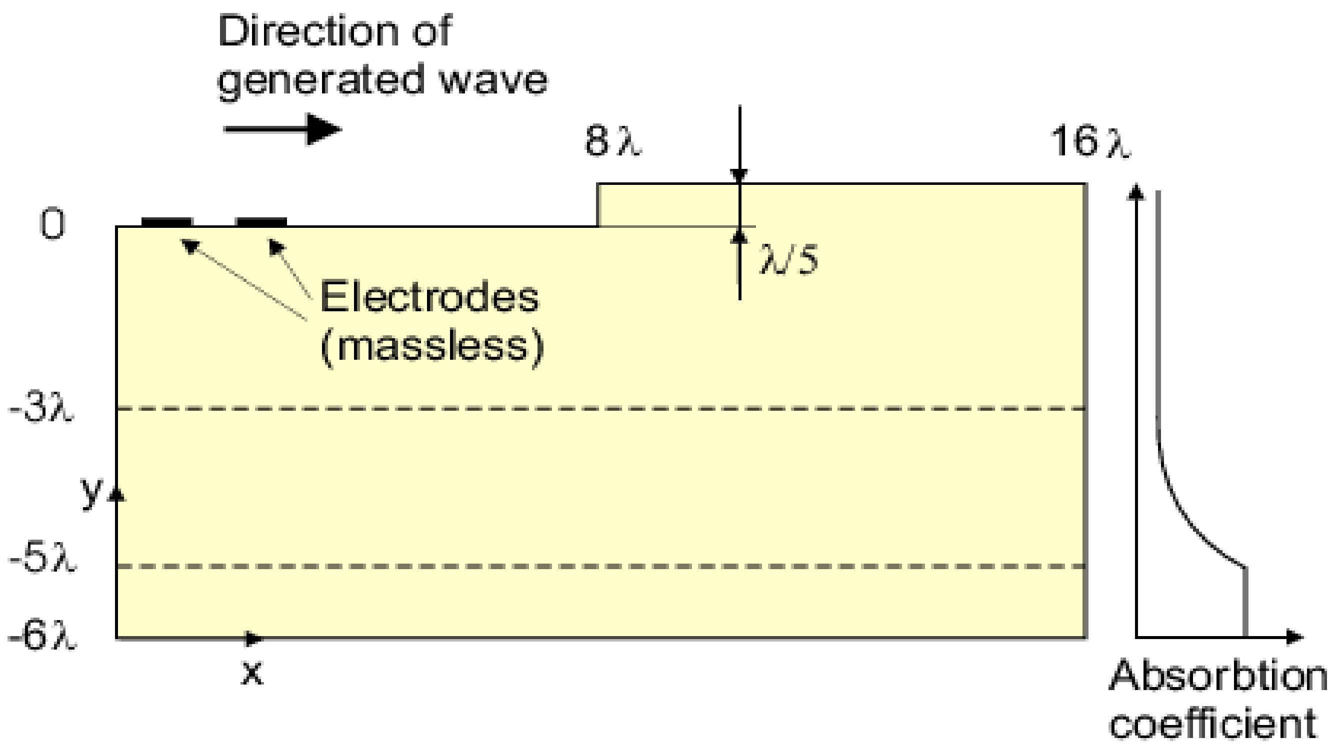
Eigenfrequency calculations of a SAW Transducer



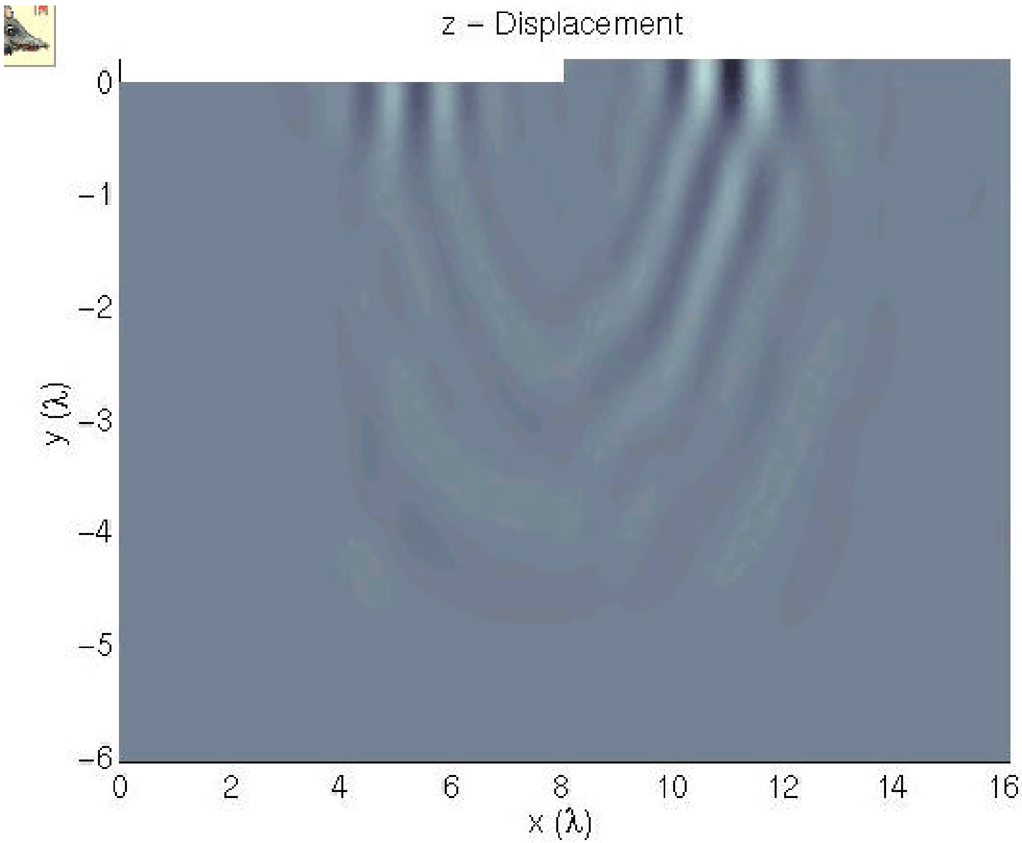


Calculated mode shape and electric potential distribution of a SAW transducer

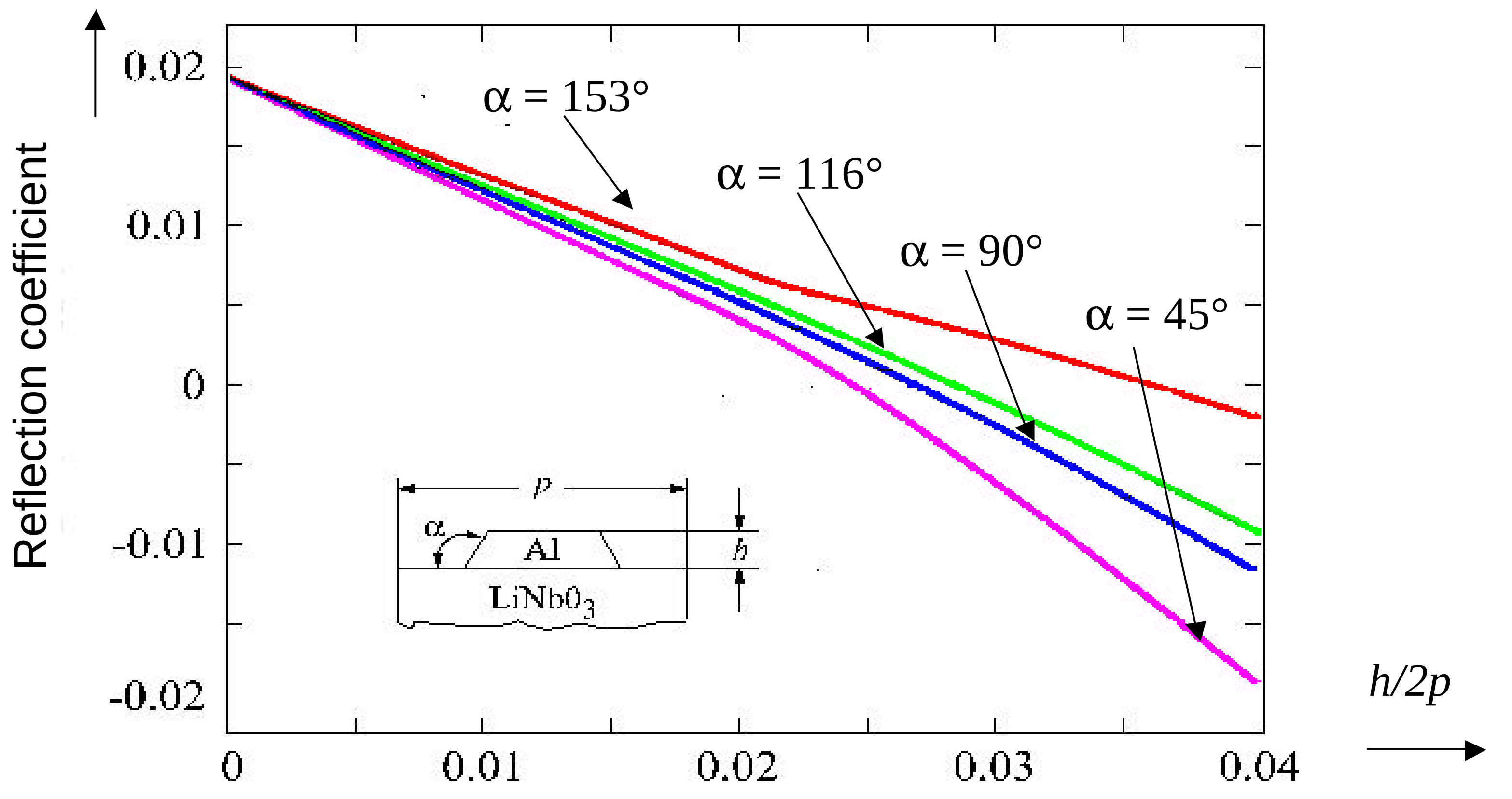
Surface wave reflected from edge



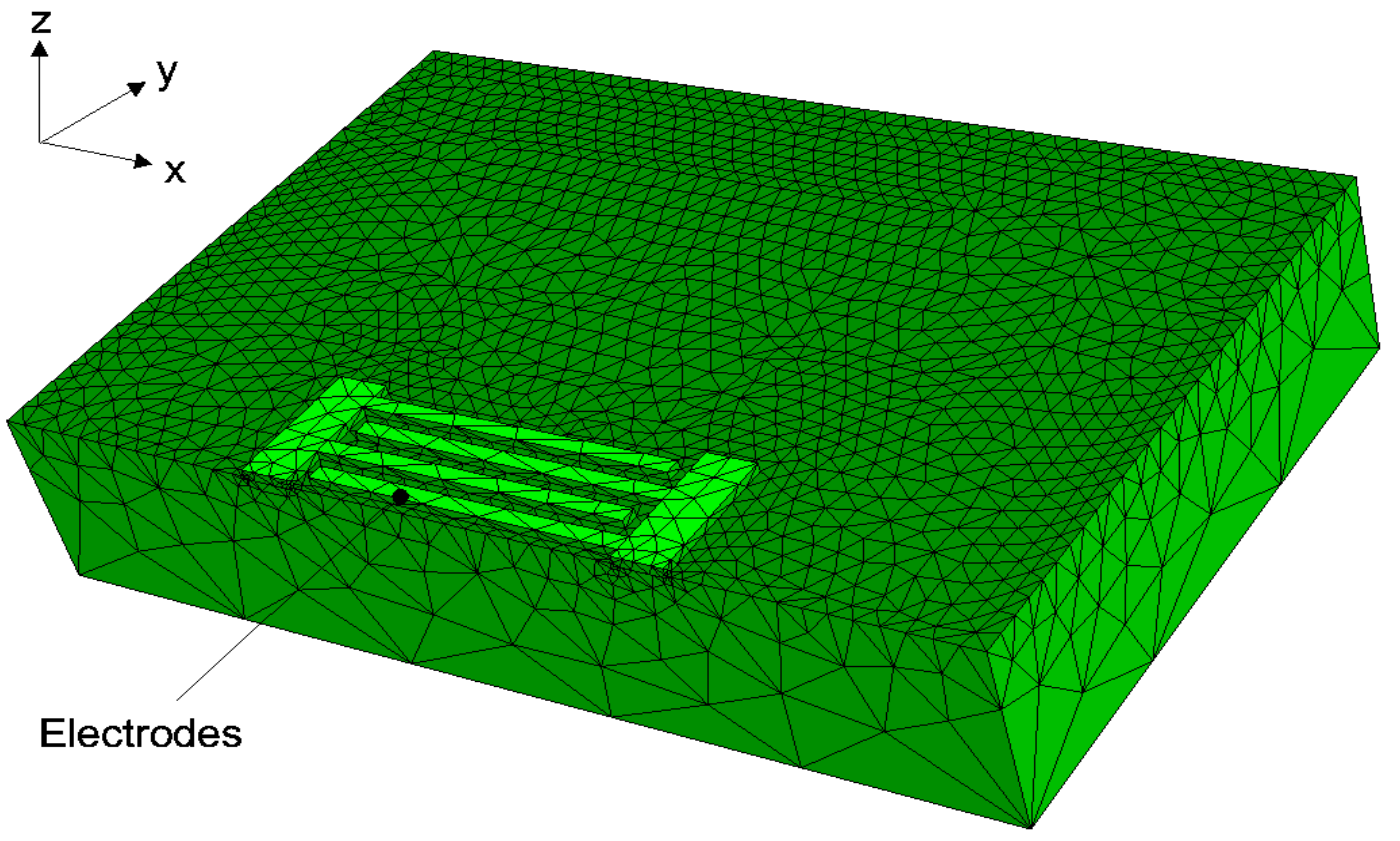
Wave reflected from edge



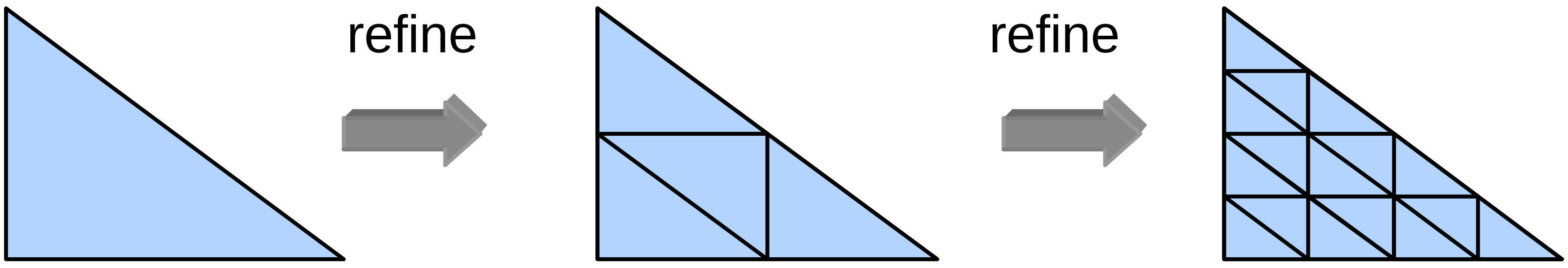
Rayleigh Wave: Reflection Coefficient at Aluminum Electrodes



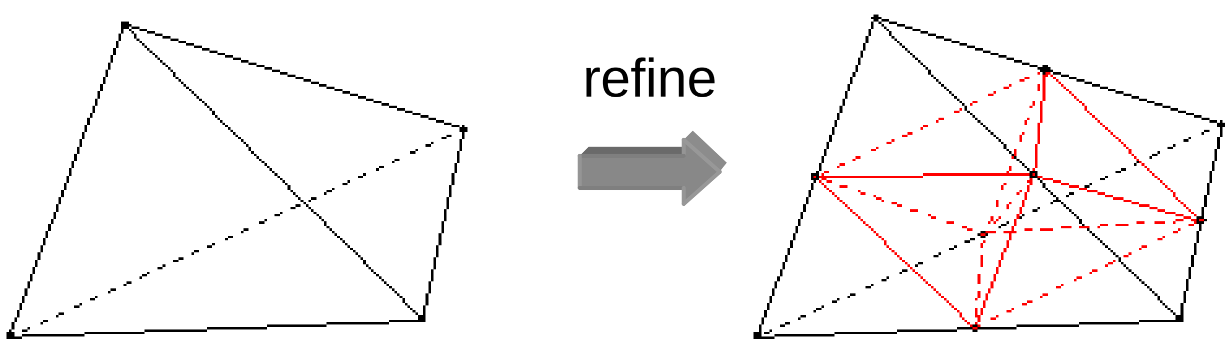
3D SAW Propagation



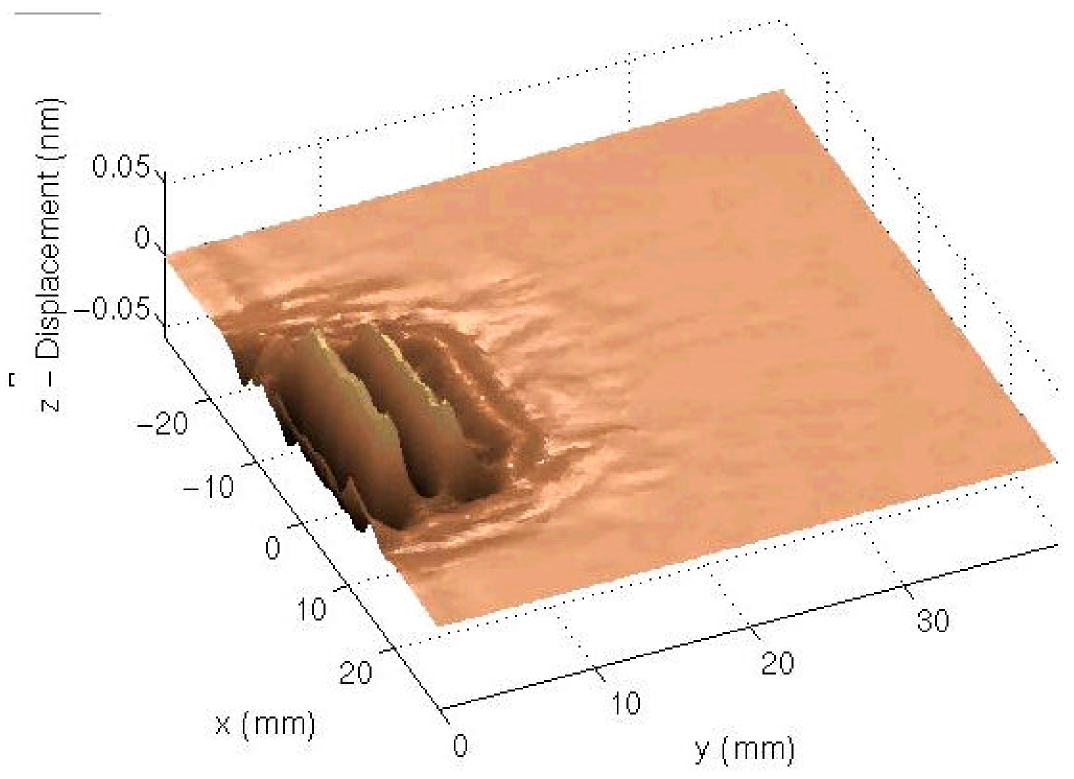
Fast Solvers via Hierarchical Grids (Multigrid)



3D:



3D SAW Propagation



Piezoceramic Multilayer Actuators

- Cofired piezoceramic multilayer actuators offer:
  - Short response time
  - High resolution and large deflection
  - Good repeatability
  - Large stiffness
- Interdigitally arranged electrodes and high driving levels lead to nonlinearities.
- Enhanced design tools needed for better insight to the occurring effects.

Need of nonlinear material modeling

Finite Element Formulation

- **Piezoelectric material parameters** are no longer treated as constant

$$\begin{aligned} \boldsymbol{T} &= \boldsymbol{c}^{\mathrm{E}}\,\boldsymbol{S} - \boldsymbol{e}^{t}\,\boldsymbol{E} \\ \boldsymbol{D} &= \boldsymbol{e}\,\boldsymbol{S} + \boldsymbol{\varepsilon}^{\mathrm{S}}\,\boldsymbol{E} \end{aligned}$$

- **Effective stiffness matrix** in finite element analysis becomes nonlinear

$$\boldsymbol{K}_h^{*}(u_h, \Phi_h)\ \{u_h, \Phi_h\} = \{F_h, Q_h\}$$

- Solutions found using **a nonlinear incremental iterative procedure**

$$\begin{aligned} \boldsymbol{K}_h^{*\,i}\,\{\Delta u_h, \Delta\Phi_h\} &= \{F_h, Q_h\} - \boldsymbol{K}_h^{*\,i}\,\{u_h^i, \Phi_h^i\} = R(u_h^i, \Phi_h^i) \\ u_h^{i+1} &= u_h^i + \eta\Delta u_h \\ \Phi_h^{i+1} &= \Phi_h^i + \eta\Delta\Phi_h \end{aligned}$$

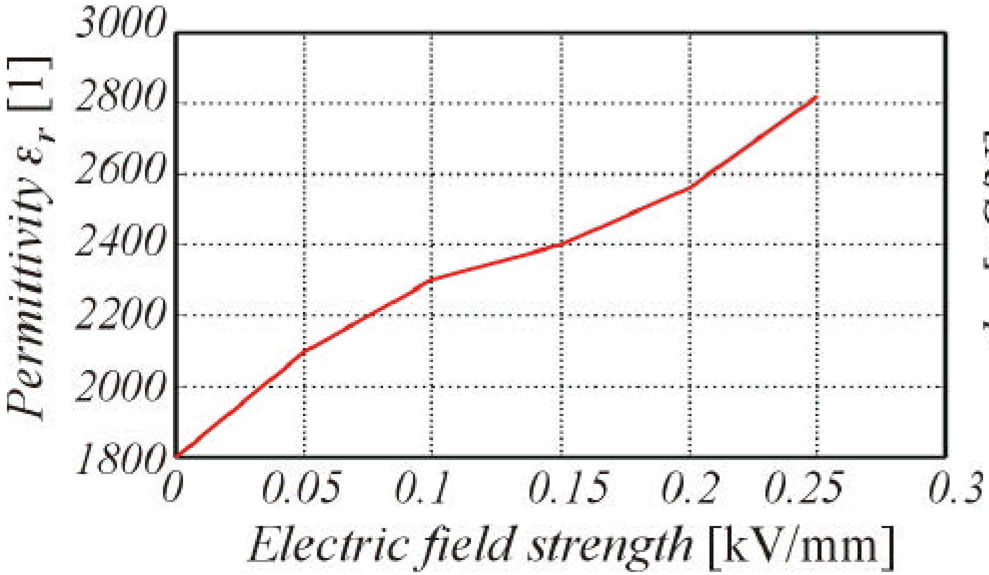
Constitutive Model

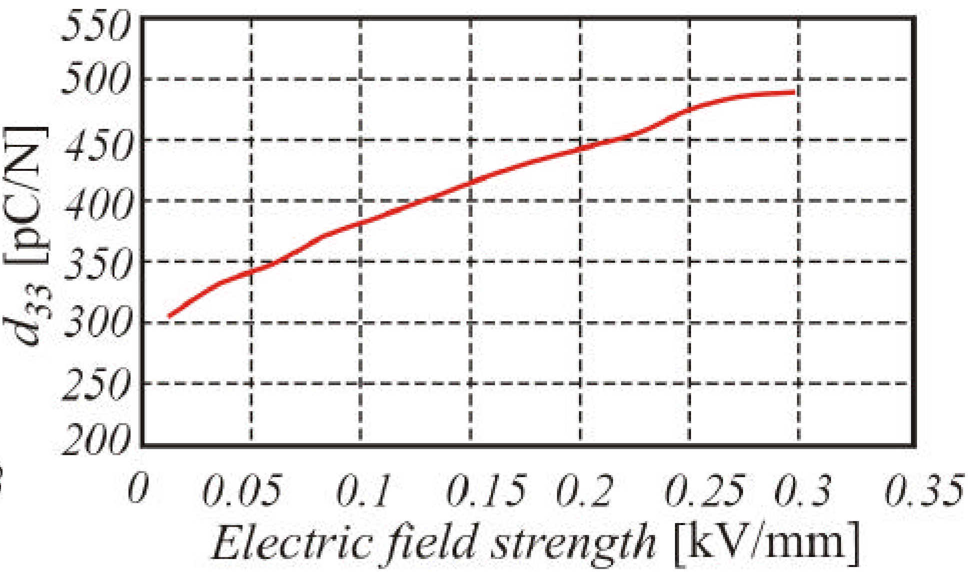
- Functional dependencies of the material constants on the current load case are included in the piezoelectric constitutive relations

	Mechanical stresses T	Electric field strength E
Modulus of elasticity	$c^E = f(T)$	
Modulus of piezoelectricity	$e = f(T,E)$	$e = f(T,E)$
Dielectric constants		$\varepsilon^S = f(E)$

Material Nonlinearities

- Measured dependencies of the material parameter on the electric field strength



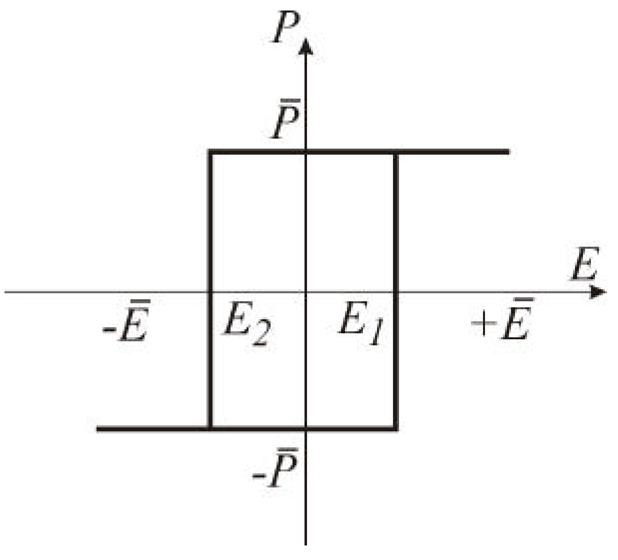


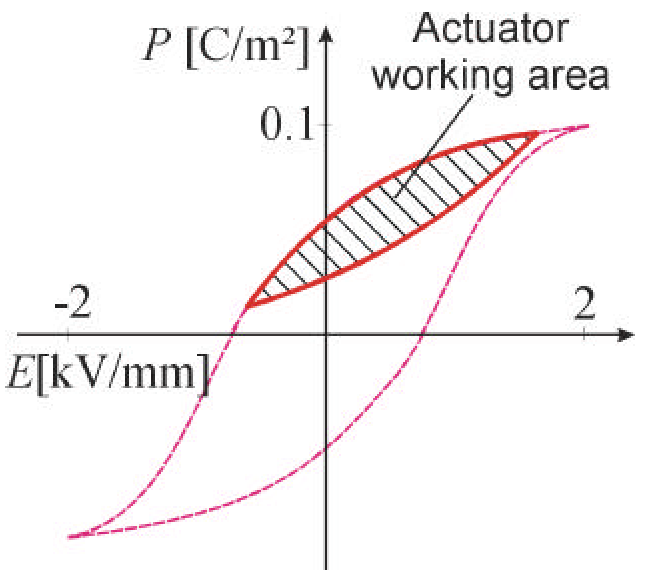
Functional dependencies are directly included in the constitutive model

Hysteresis Model

- The state of polarization is described by **a Preisach model**
- Introducing polarization in the **constitutive relations:**

$$\varepsilon^{\mathrm{S}} \boldsymbol{E} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$$

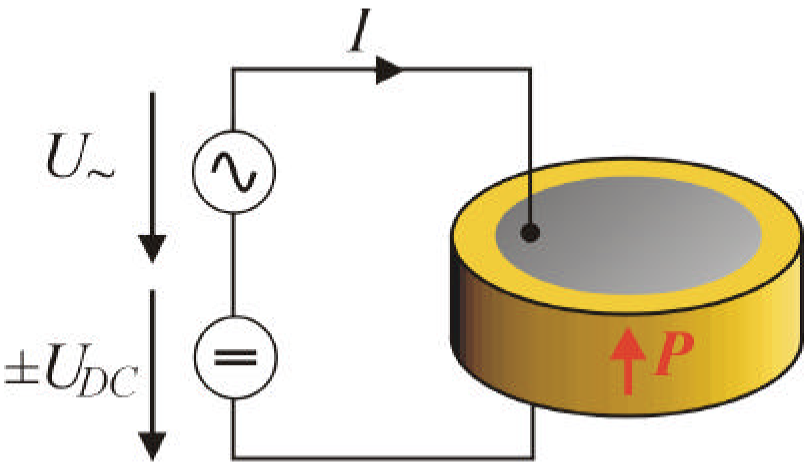




Dipole orientation given by the Preisach switching operator:

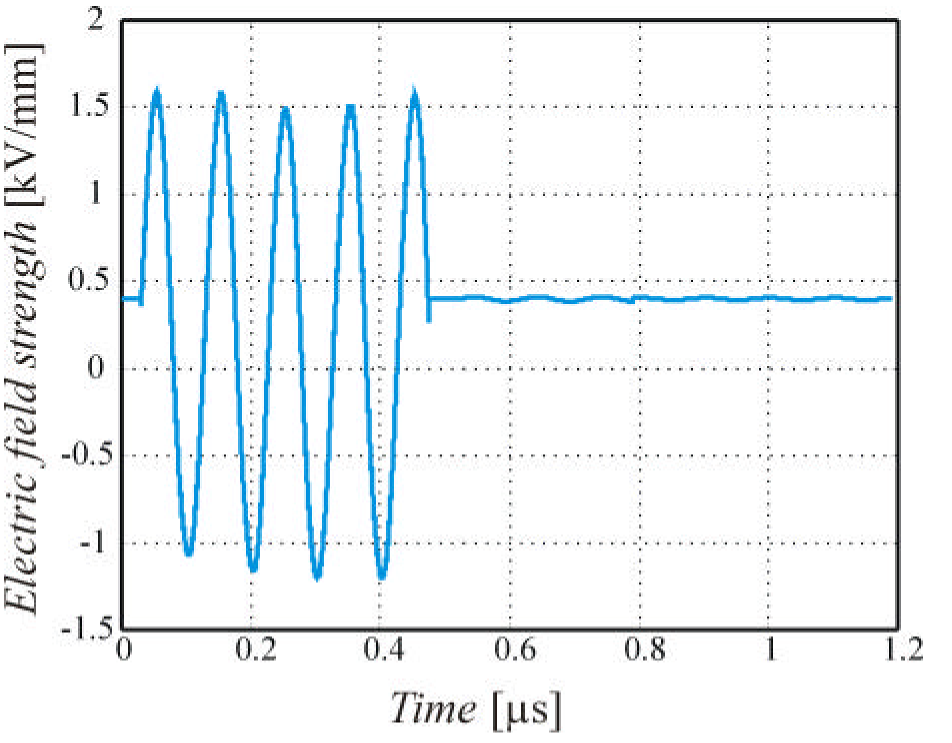
Averaging of the Preisach operator using an appropriate distribution function:

Dynamic Large Signal Behavior



Bulk ceramic transducer excited by a sinusoidal burst signal $U_{\sim}$ superimposed with a bias voltage U_{DC}

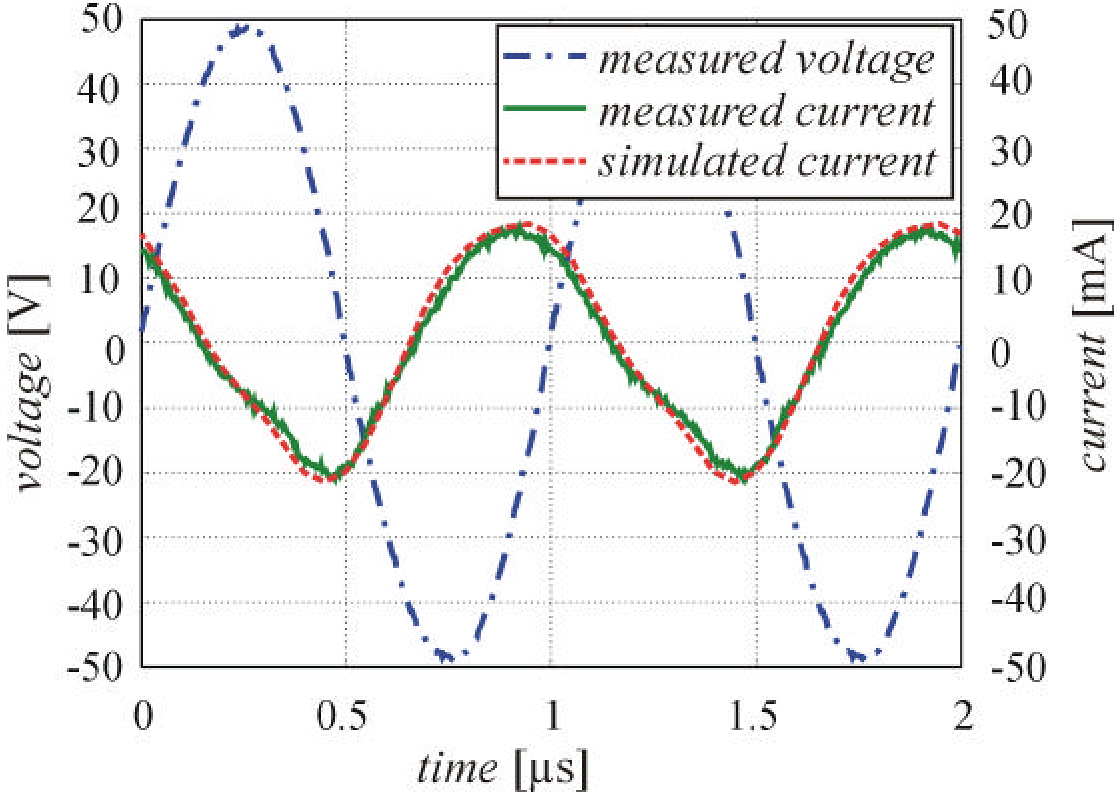
Excitation voltage sine burst

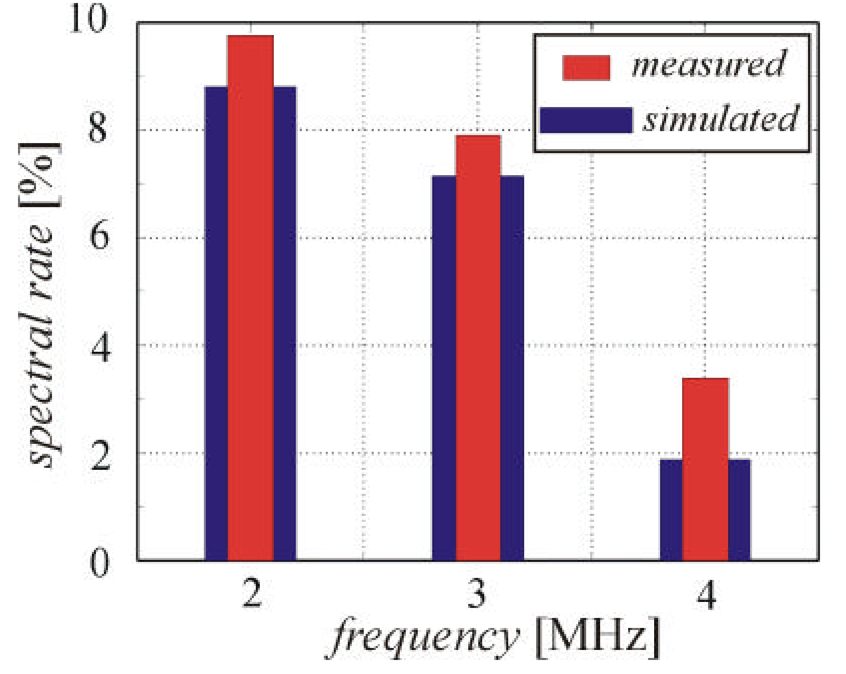


Ferroelectric hysteresis causes path dependencies and higher order harmonics in the transducers input current.

Dynamic Large Signal Excitation

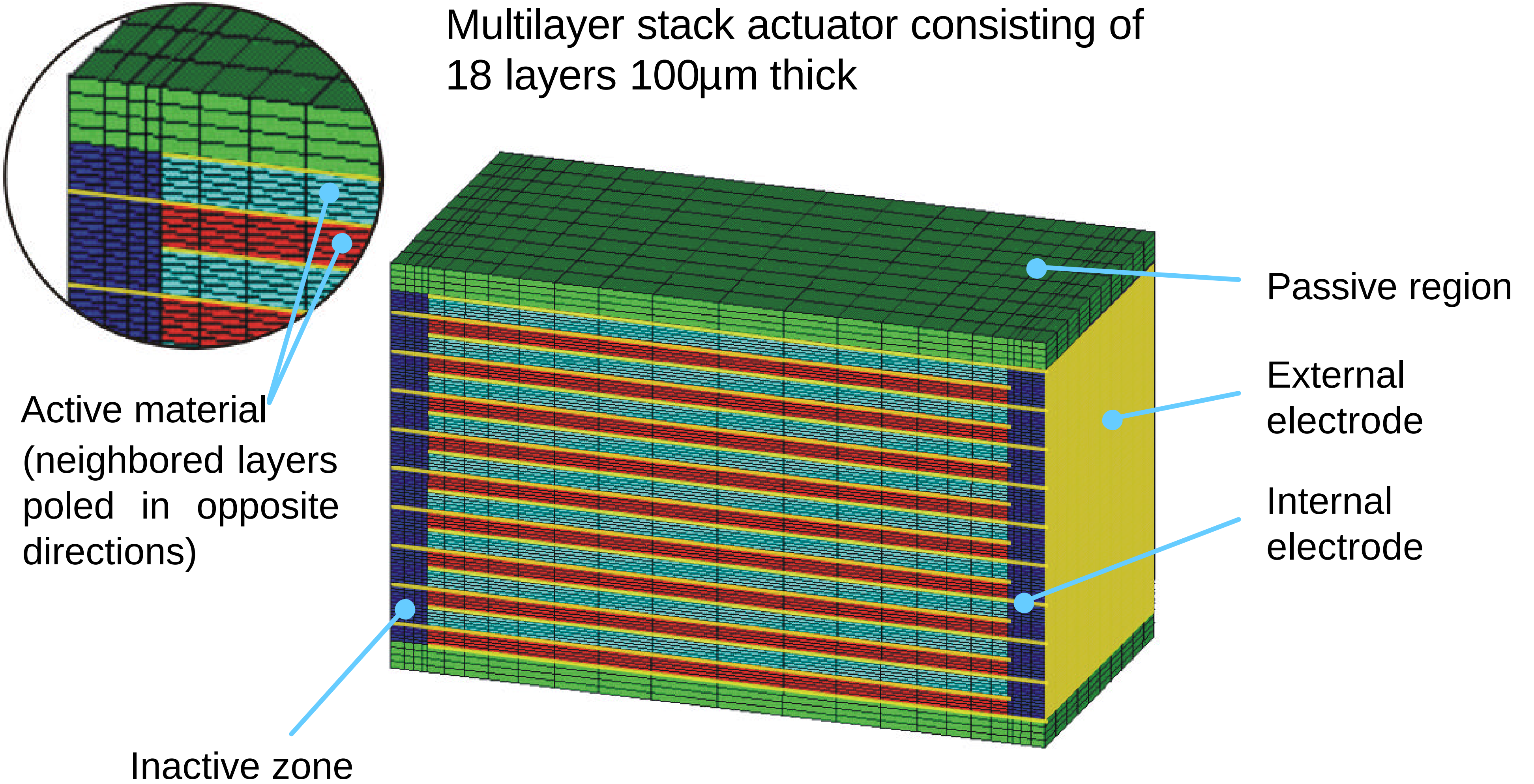
- **Ferroelectric hysteresis** effects showing up in input current





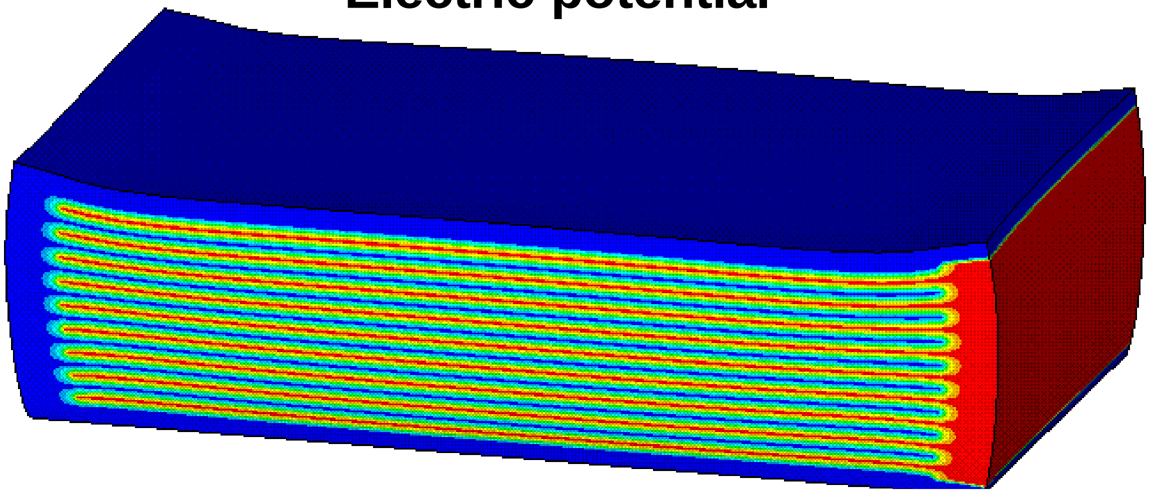
Multilayer Stack Actuator

Multilayer stack actuator consisting of 18 layers 100µm thick



Multilayer Stack Actuator

Electric potential



Electric potential

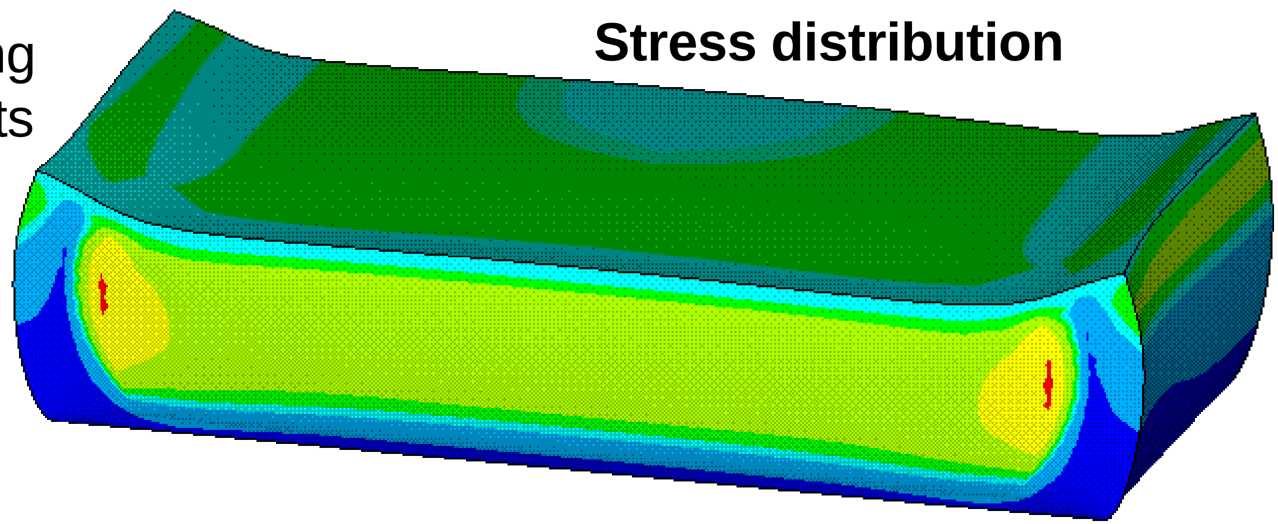
200V (max.)

0V (min.)

Numerical results performing transient analysis including ferroelectric hysteresis effects

Finite element model using 30,000 hexahedral elements

Stress distribution



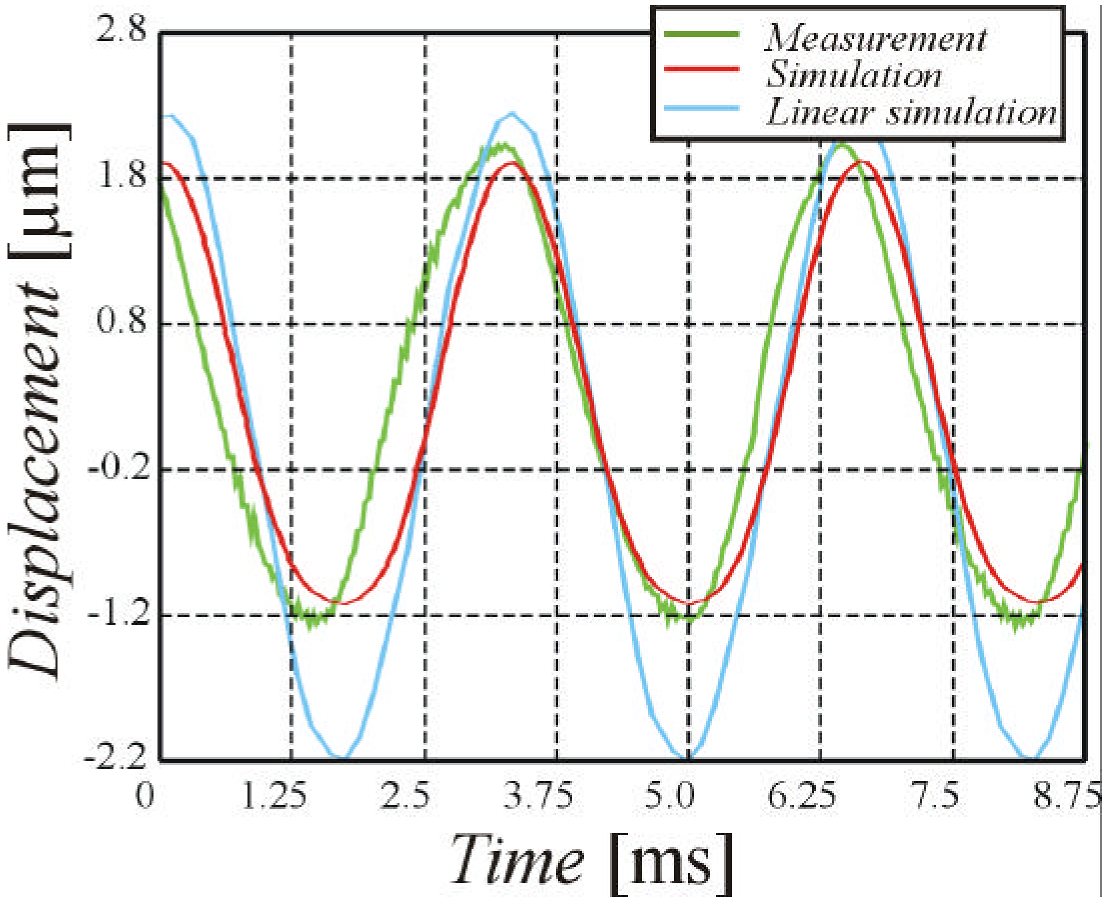
Stresses

20MPa (max.)

-20MPa (min.)

Multilayer Stack Actuator

- **Measured and simulated displacements:**

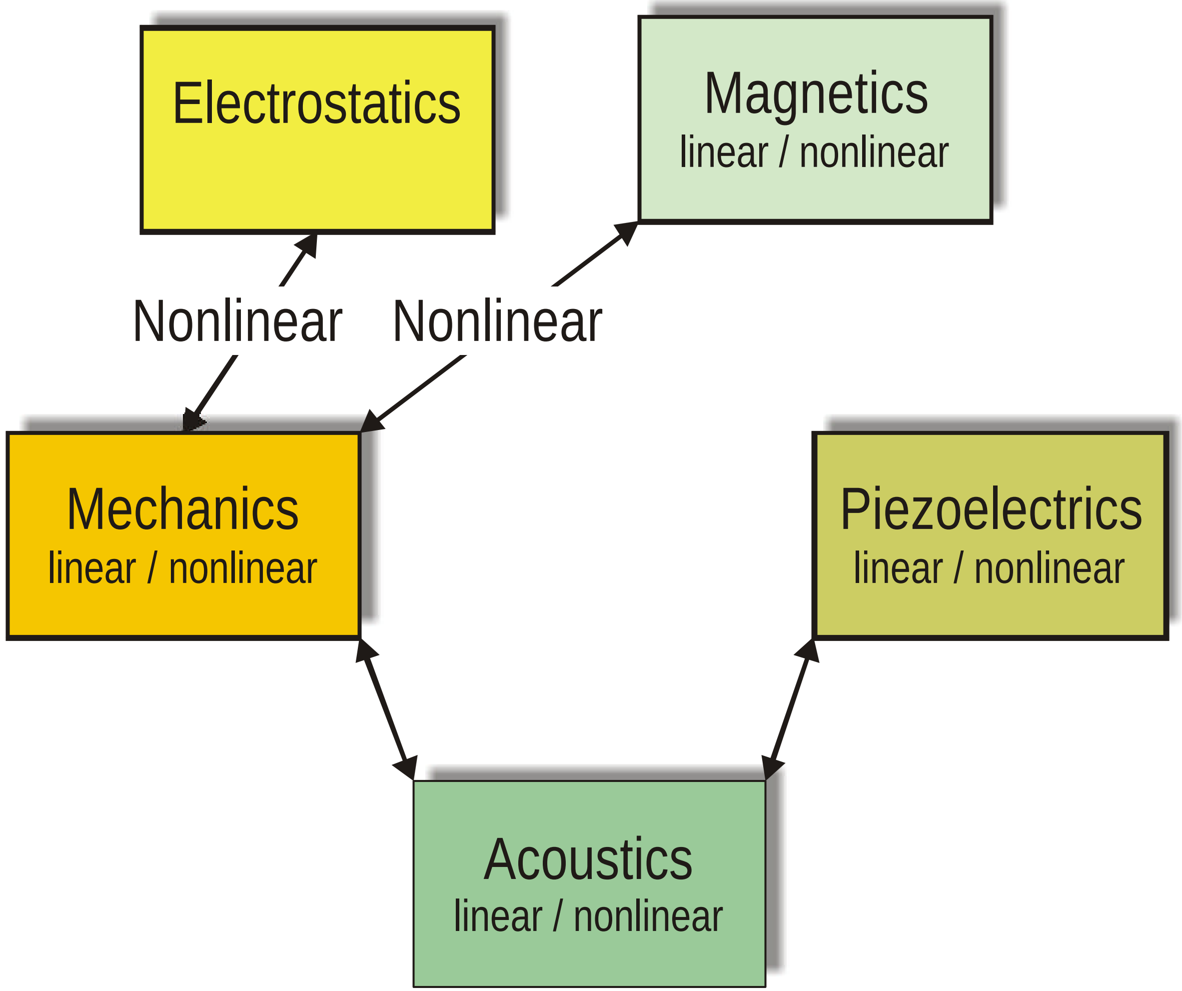


Electrostatic Transducers

- Coupling terms
  - Electrostatic force (general equation, numerical implementation)
  - Moving body in an electric field
- Iterative solution algorithm
- Voltage driven bar
  - Linear mechanical simulation
  - Nonlinear mechanical simulation
- Capacitive micromachined ultrasound transducer (CMUTs)
- Capacitive mirror actuator

Simulation of Microelectromechanical Systems (MEMS)

Coupled field problems:



Physical Fields

- Electric Field:

$$\nabla \cdot \varepsilon \nabla \phi = q$$

ϕ electric potential
q electric volume charge
ε permittivity

- Mechanical Field:

$$\mathrm{DIV}\,[\sigma] + \vec{f}_{\mathrm{V}} = \rho \frac{\partial^2 \vec{u}}{\partial t^2}$$

$[\sigma]$ stress tensor
$\vec{f}_{\mathrm{V}}$ mechanical volume force
ρ density
$\vec{u}$ mechanical displacement

- Acoustic Field:

$$\Delta \psi = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}$$

ψ velocity potential
c sound velocity

Coupling Terms

- Solid-Fluid Interface:

$$v_{\mathrm{n}} = \vec{n} \cdot \left(\frac{\partial \vec{u}}{\partial t} \right) = -\vec{n} \cdot \nabla \psi = -\frac{\partial \psi}{\partial n}$$

- Electrostatic Force:

$$\mathbf{T}_E = \begin{bmatrix} \varepsilon E_x^2 - \frac{1}{2}\varepsilon |E|^2 & \varepsilon E_x E_y & \varepsilon E_x E_z \\ \varepsilon E_y E_x & \varepsilon E_y^2 - \frac{1}{2}\varepsilon |E|^2 & \varepsilon E_y E_z \\ \varepsilon E_z E_x & \varepsilon E_z E_y & \varepsilon E_z^2 - \frac{1}{2}\varepsilon |E|^2 \end{bmatrix}$$

$$\vec{F}_E = \oint_A \mathbf{T}_E \, \vec{n} \, dS$$

Finite Element Formulation

- Semidiscrete Galerkin Formulation:

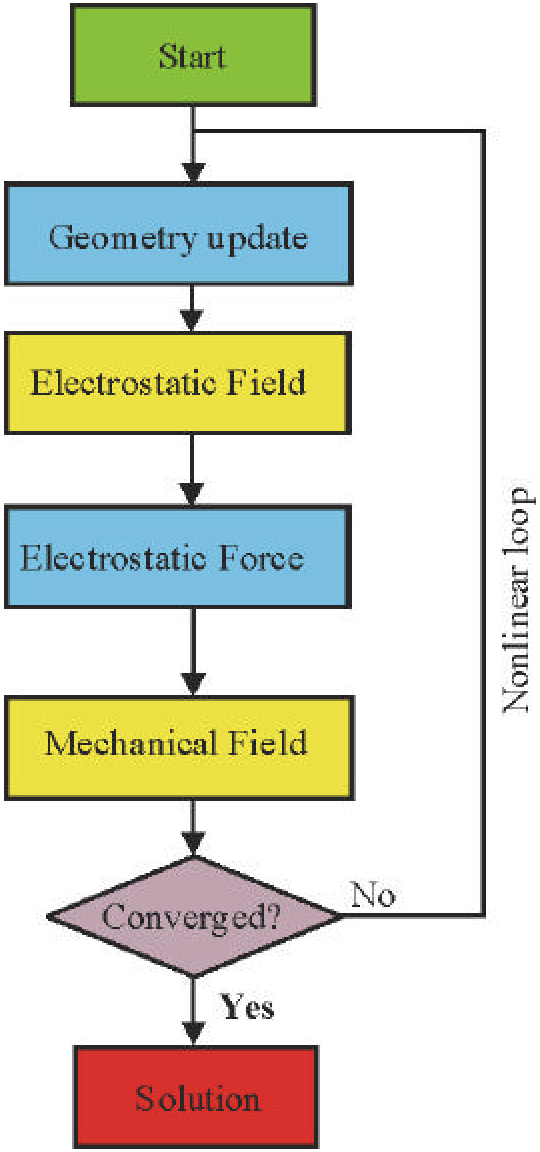
$$\begin{pmatrix} \mathbf{M}_{uu} & 0 \\ 0 & -\mathbf{M}_{\psi\psi} \end{pmatrix} \begin{pmatrix} \{\ddot{u}\} \\ \{\ddot{\Psi}\} \end{pmatrix} + \begin{pmatrix} \mathbf{C}_{uu} & \mathbf{C}_{u\psi}^T \\ \mathbf{C}_{u\psi} & -\mathbf{C}_I \end{pmatrix} \begin{pmatrix} \{\dot{u}\} \\ \{\dot{\Psi}\} \end{pmatrix} + \begin{pmatrix} \mathbf{K}_{uu} & 0 \\ 0 & -\mathbf{K}_{\psi} - \mathbf{K}_I \end{pmatrix} \begin{pmatrix} \{u\} \\ \{\Psi\} \end{pmatrix} = \begin{pmatrix} \{F_u(\phi)\} \\ \{0\} \end{pmatrix}$$

$$\mathbf{K}_\phi(u)\{\Phi\} = \{F_\phi(u)\}$$

- Algebraic Equation:

$$\begin{pmatrix} \mathbf{M}_{uu}^* & \mathbf{C}_{u\psi}^T & 0 \\ \mathbf{C}_{u\psi} & \mathbf{M}_{\psi\psi}^* & 0 \\ 0 & 0 & \mathbf{K}_\phi(u_i^{n+1}) \end{pmatrix} \begin{pmatrix} \{u_{i+1}^{n+1}\} \\ \{\Psi_{i+1}^{n+1}\} \\ \{\Phi_{i+1}^{n+1}\} \end{pmatrix} = \begin{pmatrix} \{F_u(\phi_i^{n+1})\} \\ \{0\} \\ \{F_\phi(u_i^{n+1})\} \end{pmatrix}$$

Iterative Solution Algorithm

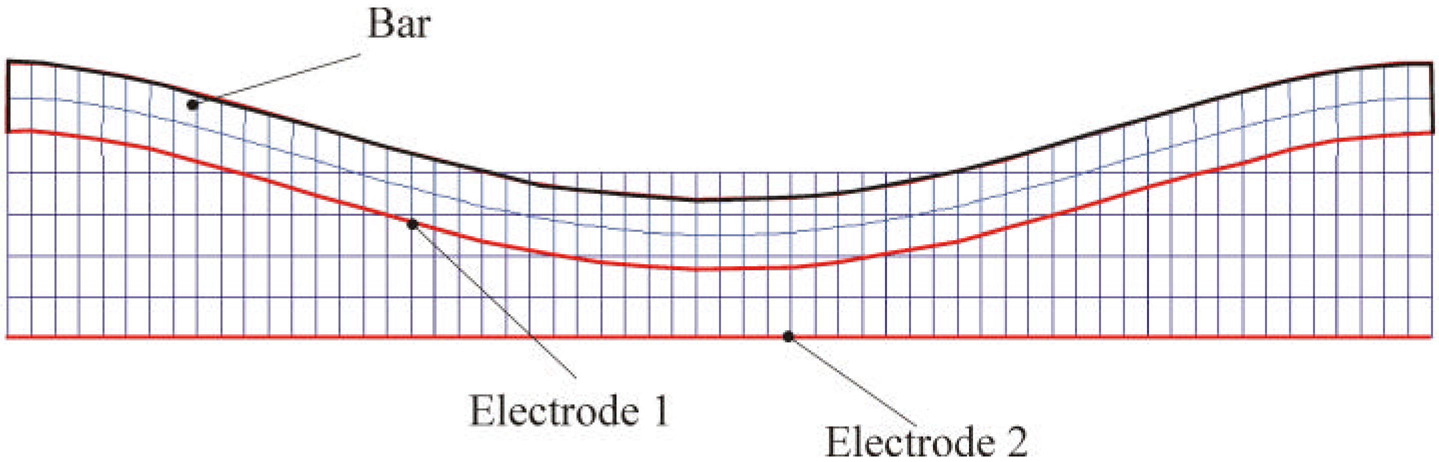


- Convergence test:

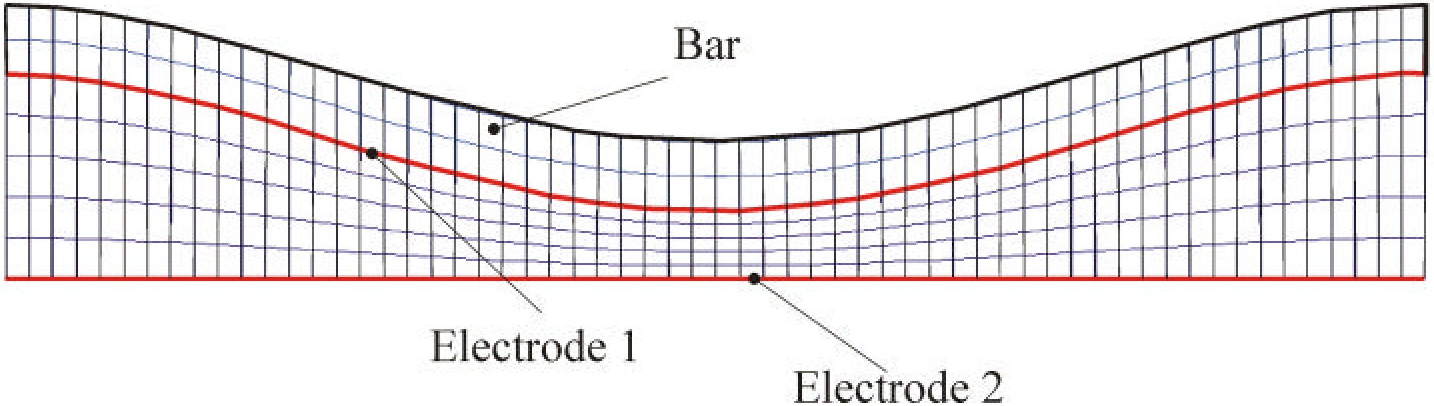
$$\frac{||u_{i+1}^{n+1} - u_i^{n+1}||_2}{||u_{i+1}^{n+1}||_2} < \delta_i$$

Moving Body in an Electric Field

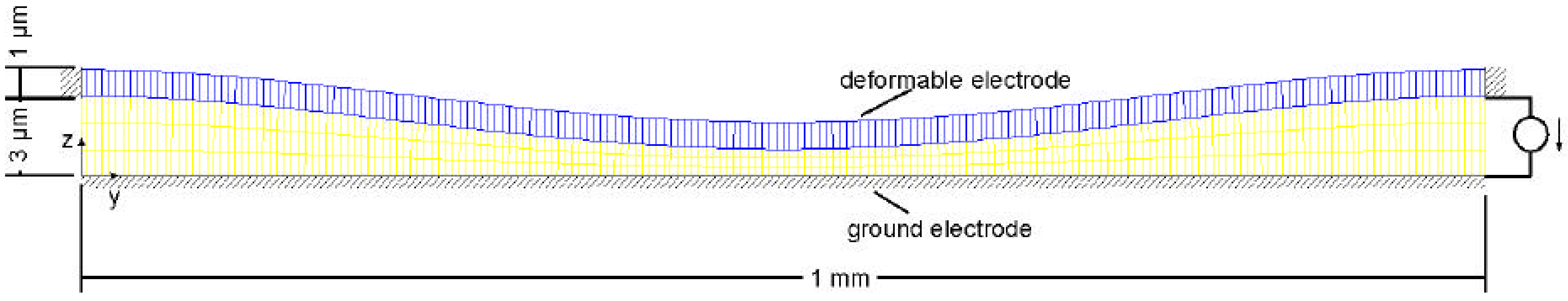
- Standard:



- Moving mesh technique:

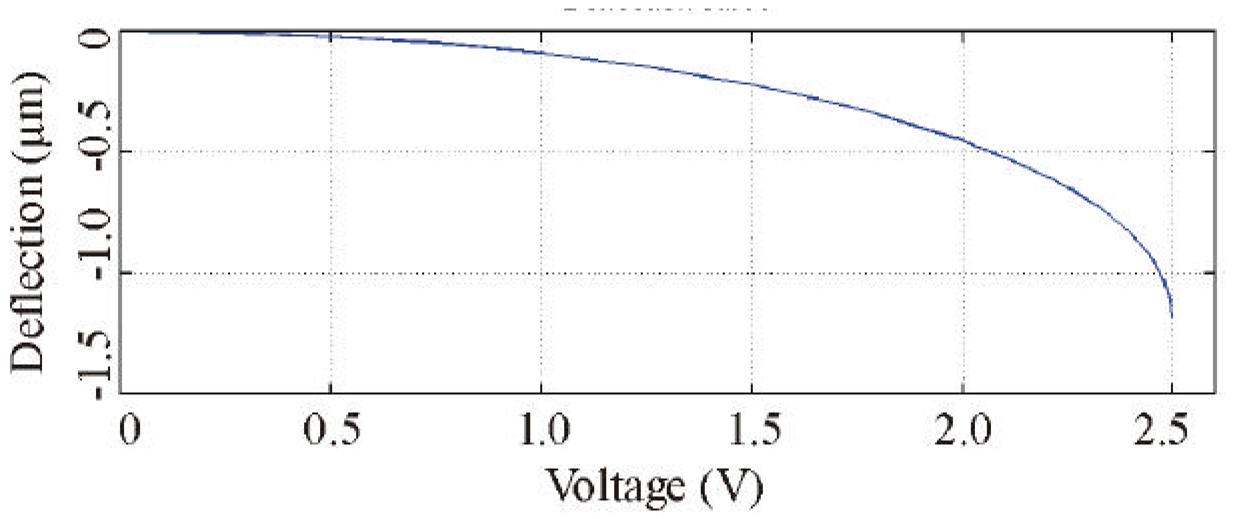


Voltage Driven Bar (I)

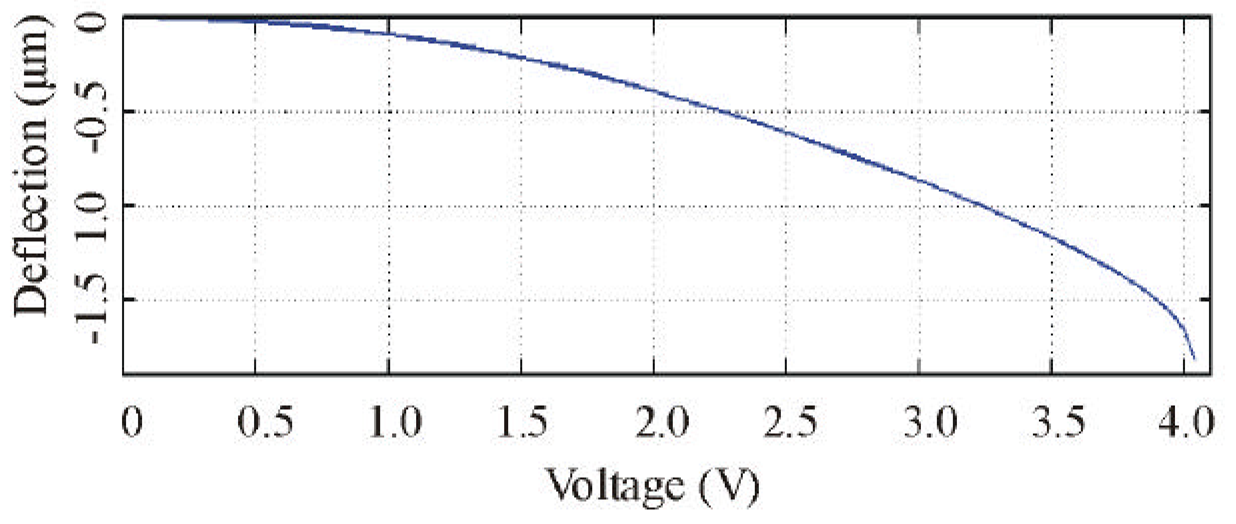


- Full coupling between mechanics and electrostatics
- Snap-In effekt
- Dimension: 1 mm x 1 µm; 3 µm Gap

Voltage Driven Bar (II)

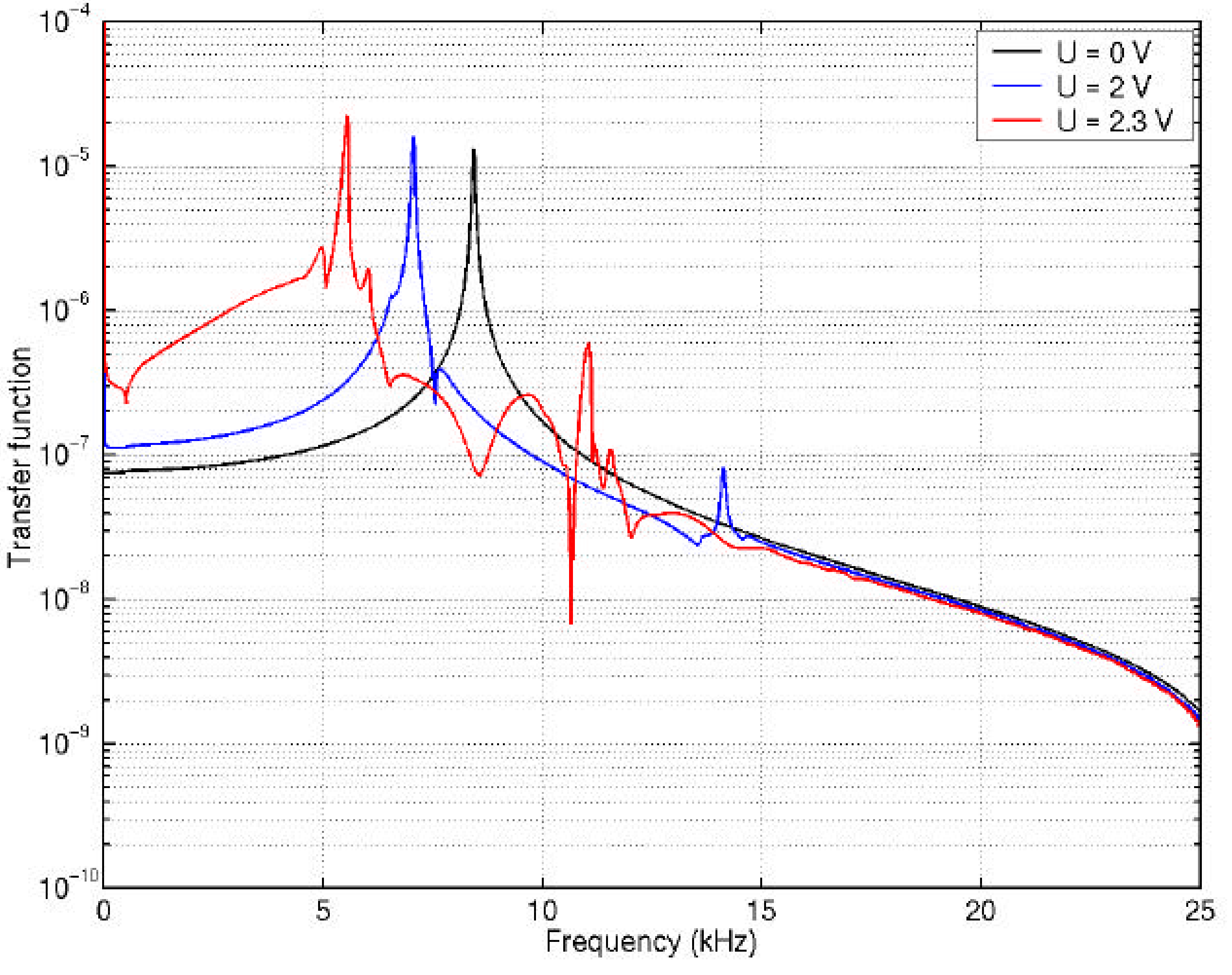


Mechanical linear case



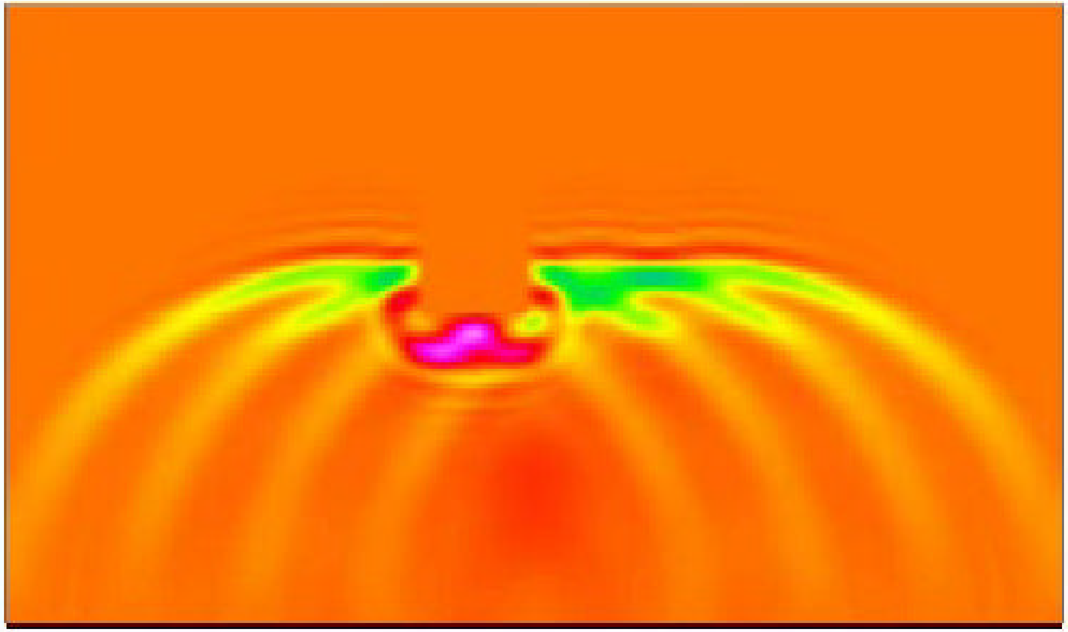
Mechanical nonlinear case

Freqency Spectrum

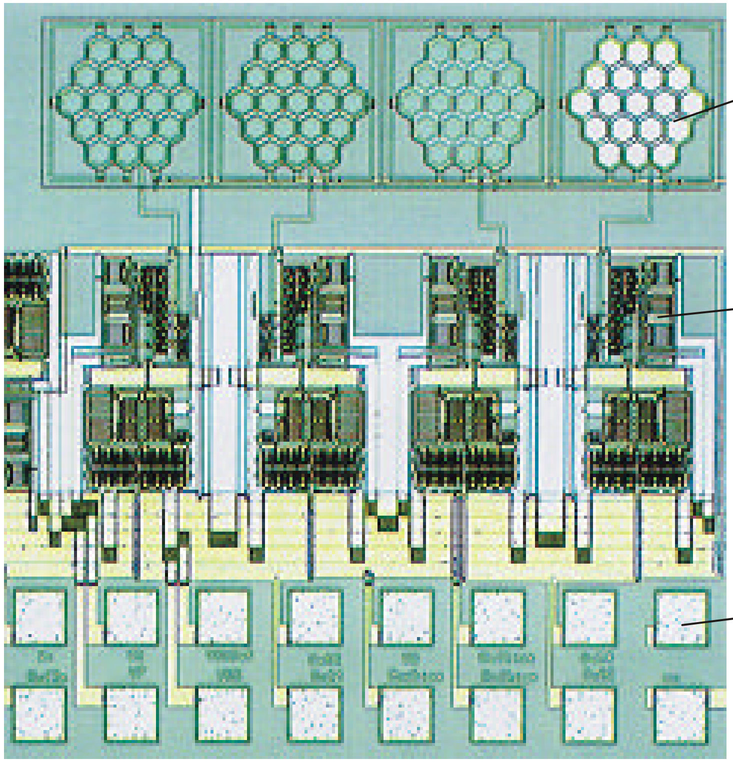


Transducer Array: Puls Echo Mode

- Array with 4 Transducers
- Barricade over Transducer #2



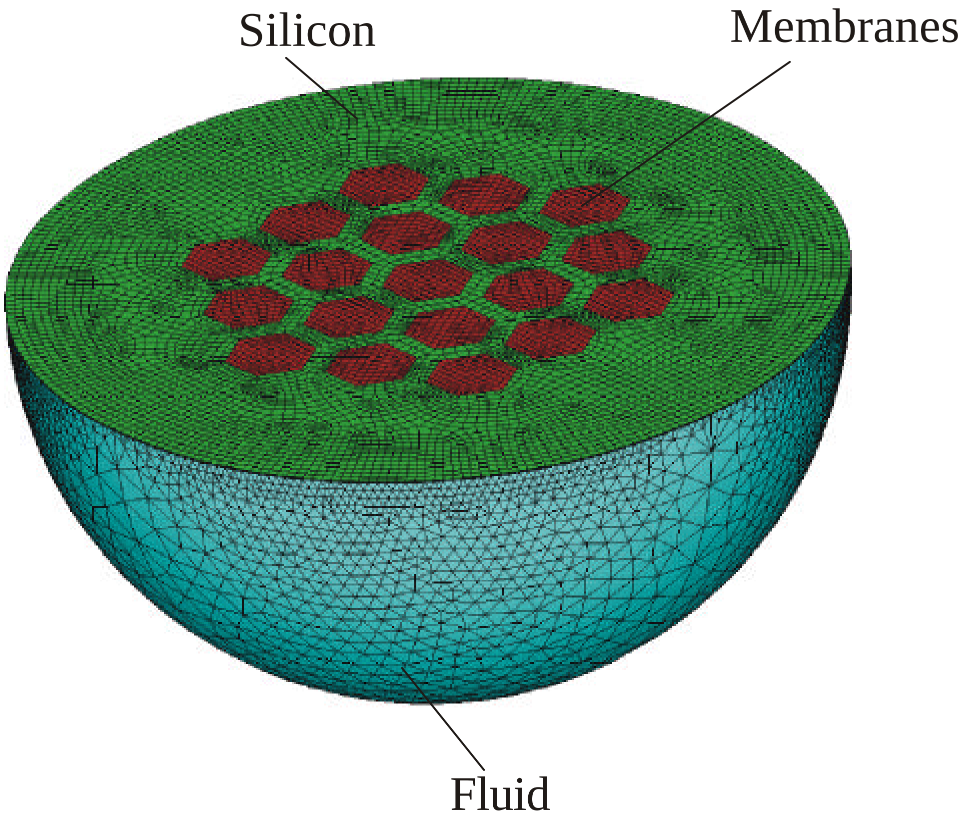
Micromachined Capacitive Ultrasound Array



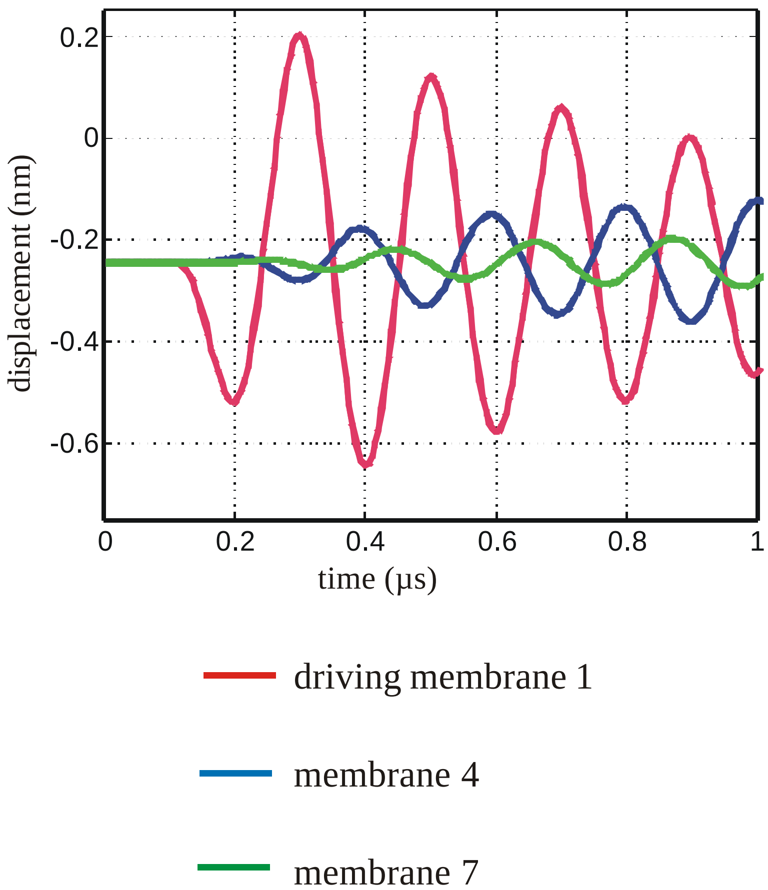
Experiments showed:

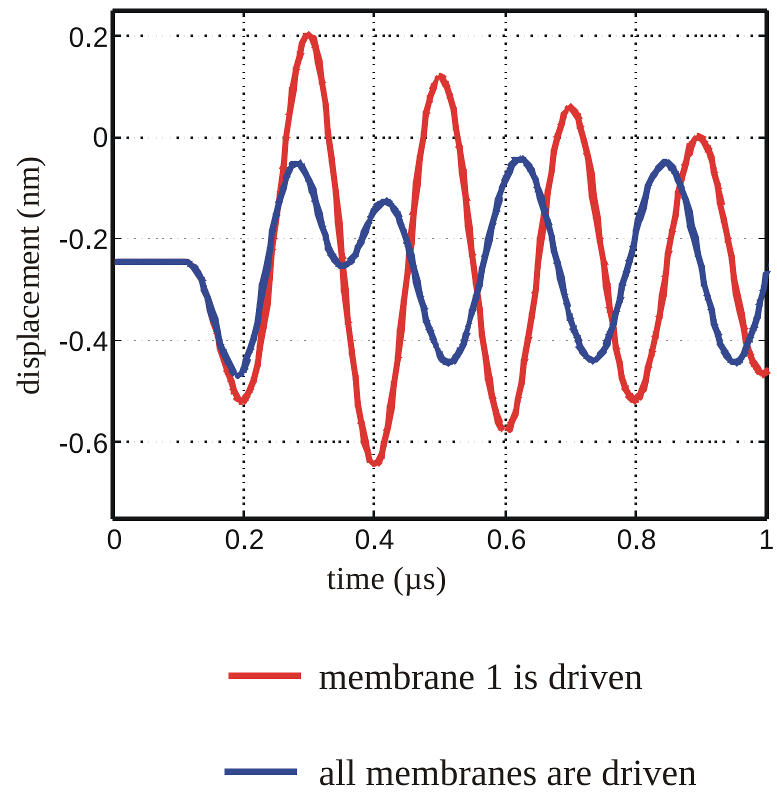
- Long ring down time of membrane deflections
- Strong cross talk between individual membrane

Finite Element Model



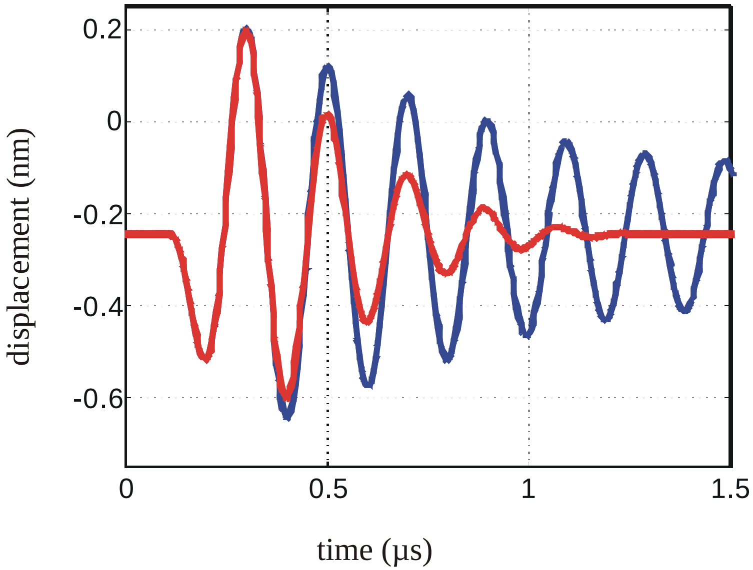
Cross Talk: Uncontrolled Membranes





Controlled Membranes

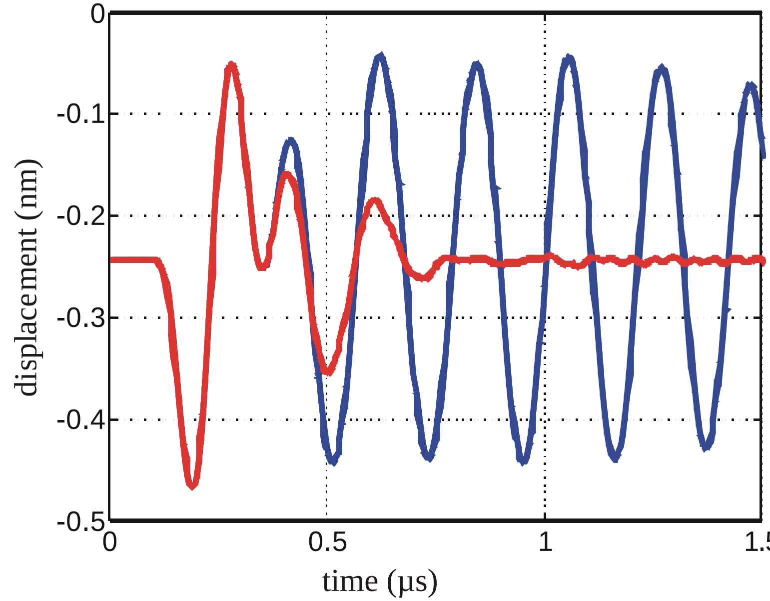
Membrane 1 excited



controlled membrane 2

uncontrolled membrane 2

All Membranes excited

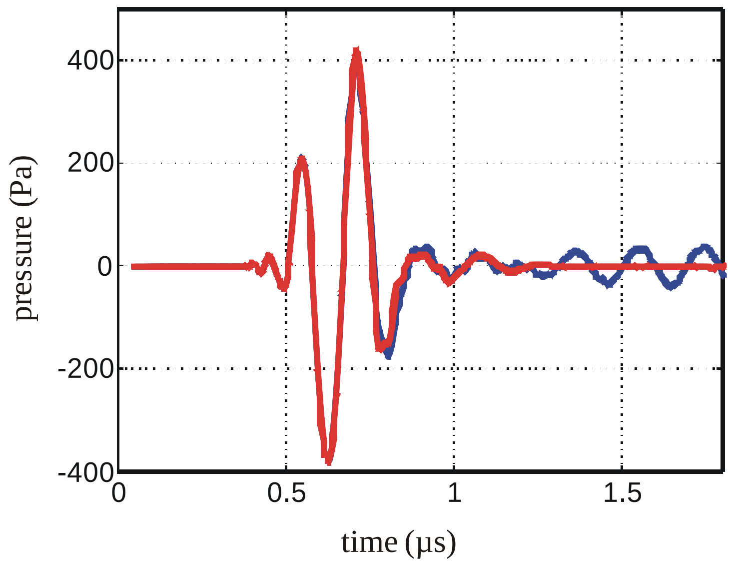


controlled membrane 1

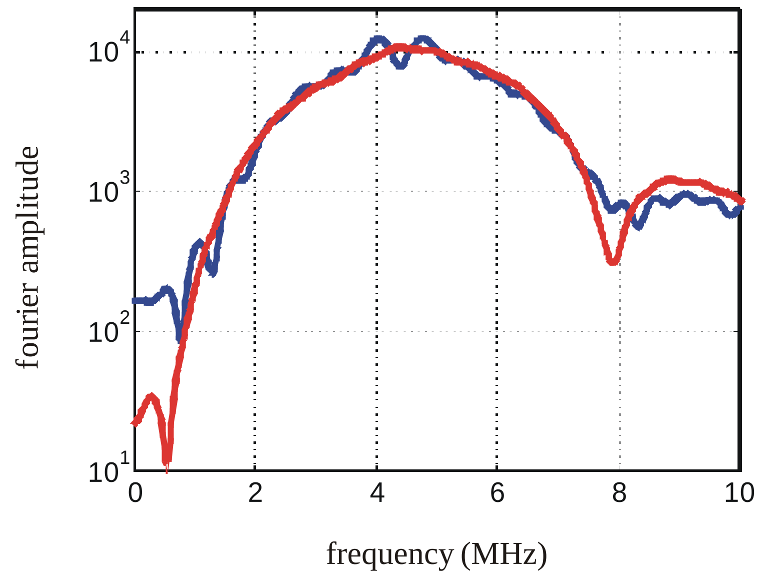
uncontrolled membrane 1

Controlled array

Pressure Signal



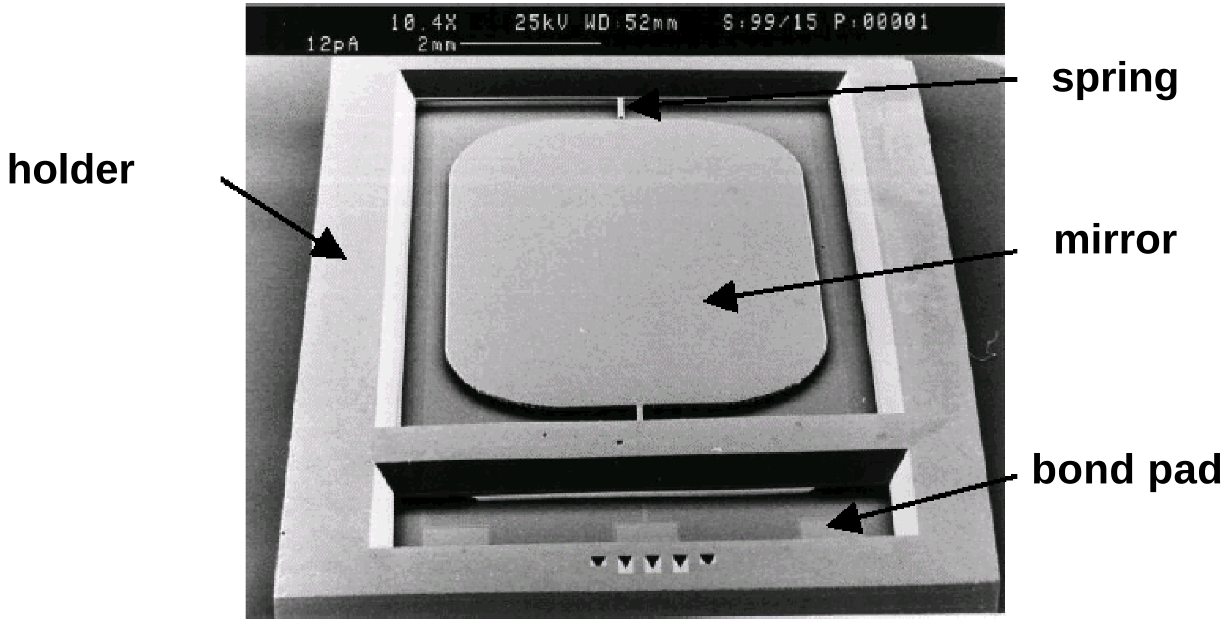
Frequency Spectrum



controlled array (all membranes excited)

uncontrolled array (all membranes excited)

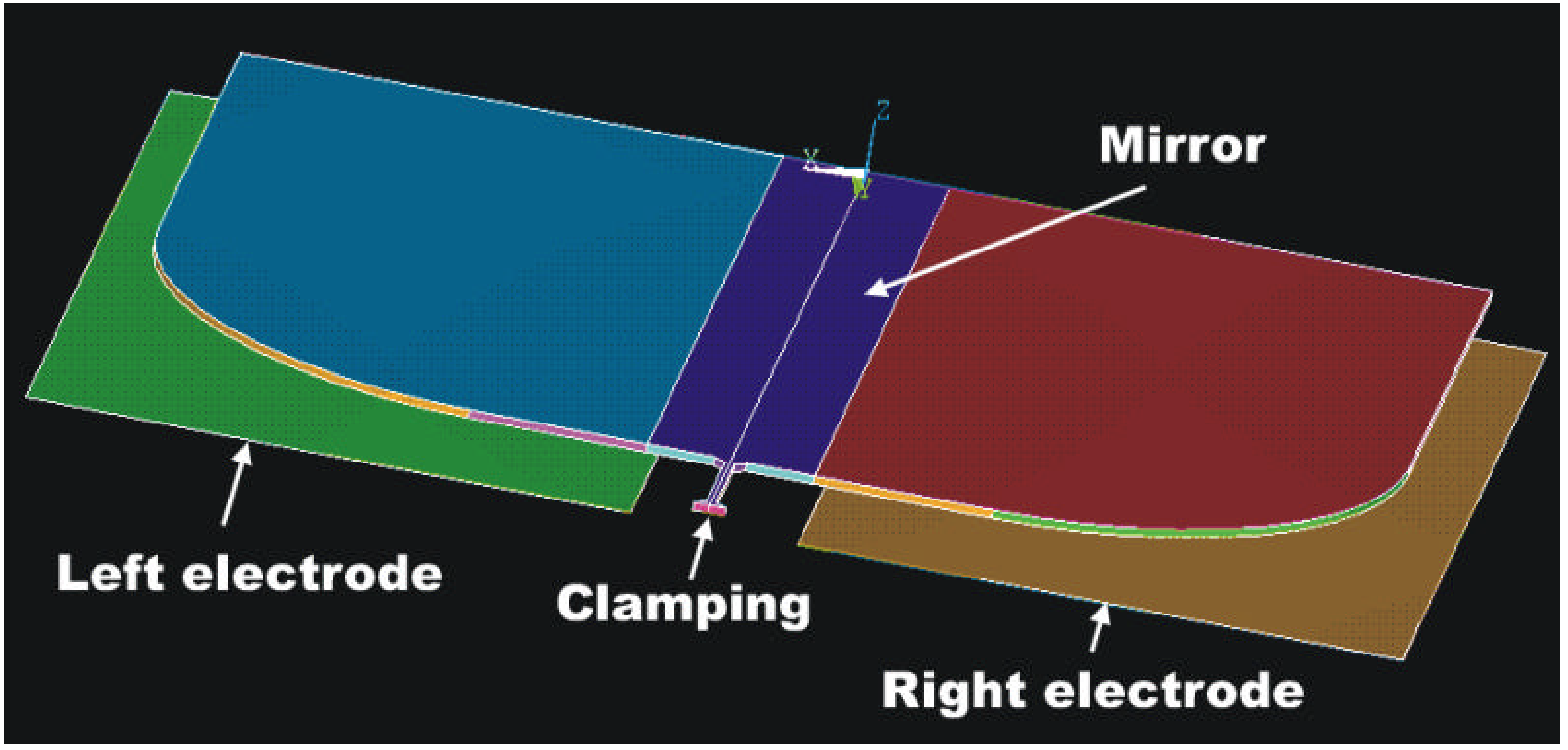
3D Simulation of a Tip Mirror



- Design, implementation and measuring results (TU Chemnitz): Long ring down time of membrane deflections
- Dimensions:
  - – 6000 x 9000 x 40 µm³
  - – 211 µm electrode gap

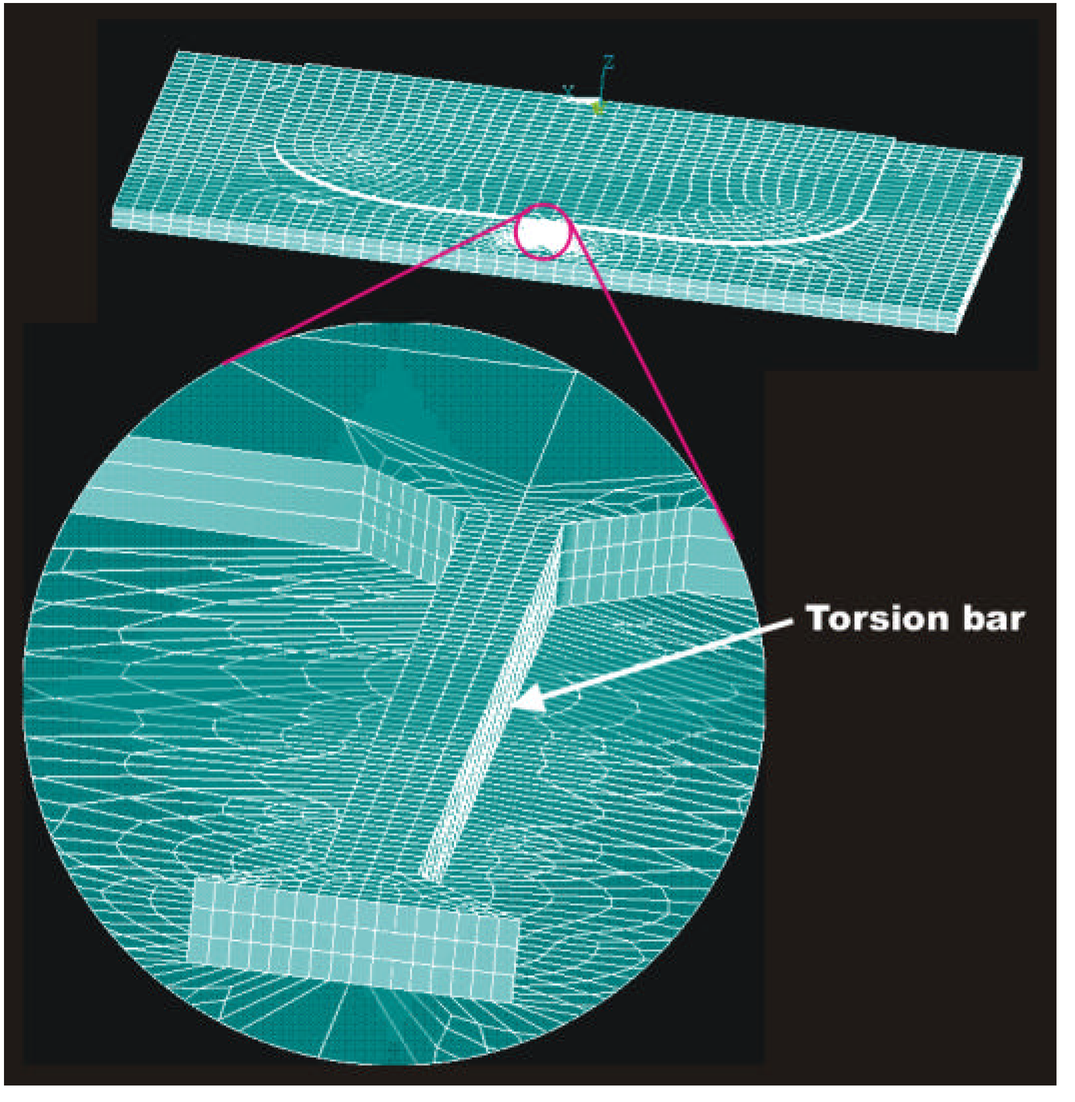
3D Simulation of a Tip Mirror

- Model:



3D Simulation of a Tip Mirror

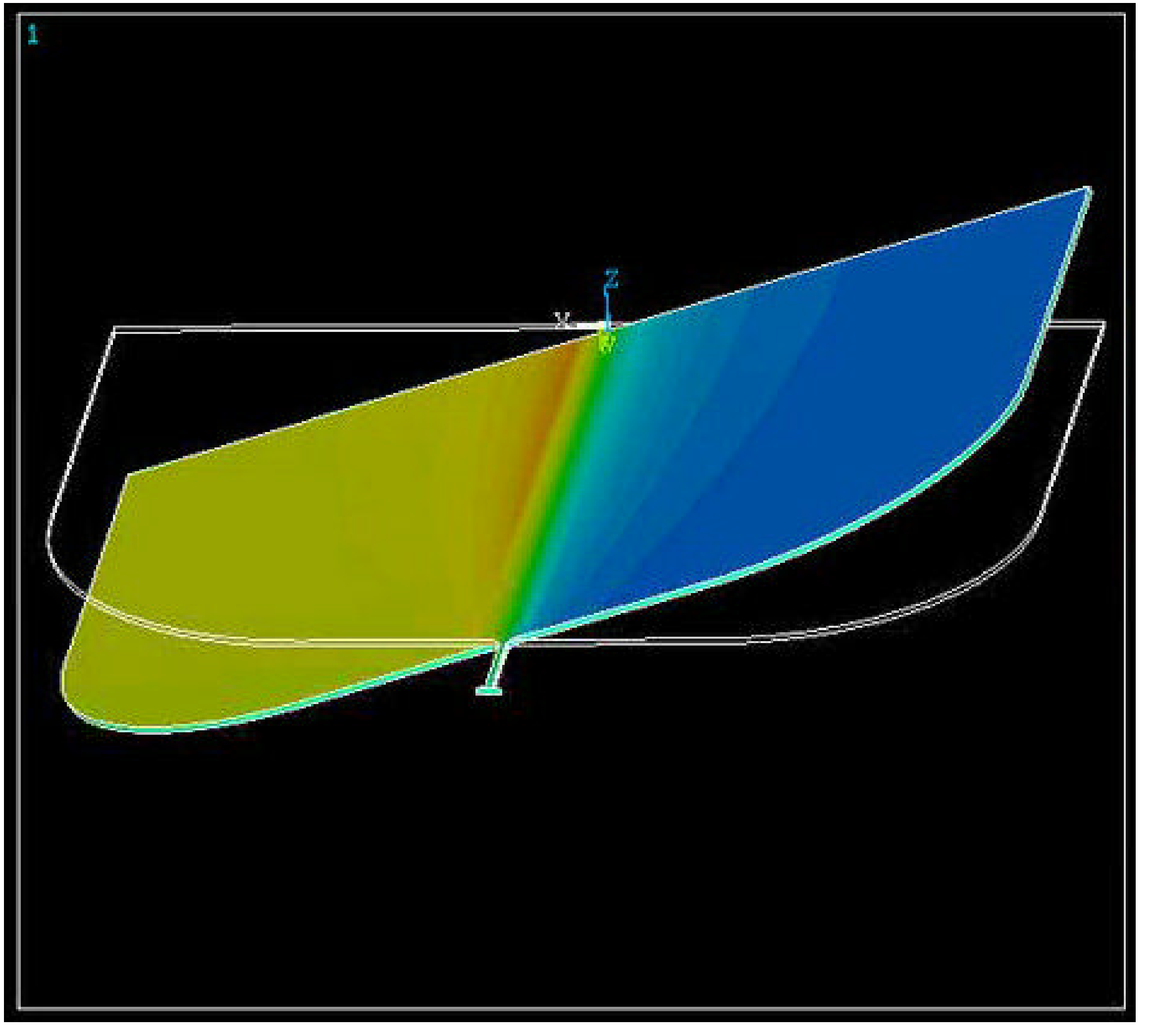
- **Meshing:**



3D Simulation of a Tip Mirror

- **Jump response:**

Contour values show bending deformation of mirror.



Magnetomechanical Transducers

- Magnetic field computation
- Eddy current sensor
  - FE-model and domain discretization as a function of penetration depth
- Coupling terms
  - Electromagnetic force (general equation, numerical implementation)
  - Moving body in a magnetic field
- Applications
  - Electromagnetic acoustic transducer (EMAT)
  - Electrodynamic loudspeaker
  - Sound emission of loaded power transformer
  - Electromagnetic valve

Magnetic Field Equations (I)

- **Maxwell‘s equations** for eddy current problems:

$$
\begin{aligned}
\nabla \times \mathbf{H} &= \mathbf{J} \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{B} &= 0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{H} &= \nu(B)\mathbf{B} \\
\mathbf{J} &= \gamma \mathbf{E}
\end{aligned}
$$

$\mathbf{H}$	magnetic field intensity
$\mathbf{J}$	total electric current density
$\mathbf{E}$	electric field intensity
$\mathbf{B}$	magnetic induction
ν	magnetic reluctivity
γ	electric conductivity

- Boundary condition:

$$\mathbf{B} \cdot \vec{n} = 0$$

- Interface conditions:

$$
\begin{aligned}
[\mathbf{B} \cdot \mathbf{n}] &= \mathbf{B}_i \cdot \mathbf{n} - \mathbf{B}_j \cdot \mathbf{n} = 0 \\
[\mathbf{H} \times \mathbf{n}] &= \mathbf{H}_i \times \mathbf{n} - \mathbf{H}_j \times \mathbf{n} = 0
\end{aligned}
$$

Problem Formulation (II)

- Introducing the magnetic **vector potential** by $\mathbf{B} = \nabla \times \mathbf{A}$ results in:

$$\nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0 \quad \Longrightarrow \quad \mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}$$

- Partial Differential Equation:

$$\gamma \frac{\partial \mathbf{A}}{\partial t} + \nabla \times \nu \nabla \times \mathbf{A} = \mathbf{J}_{\mathrm{i}}$$

- Boundary condition:

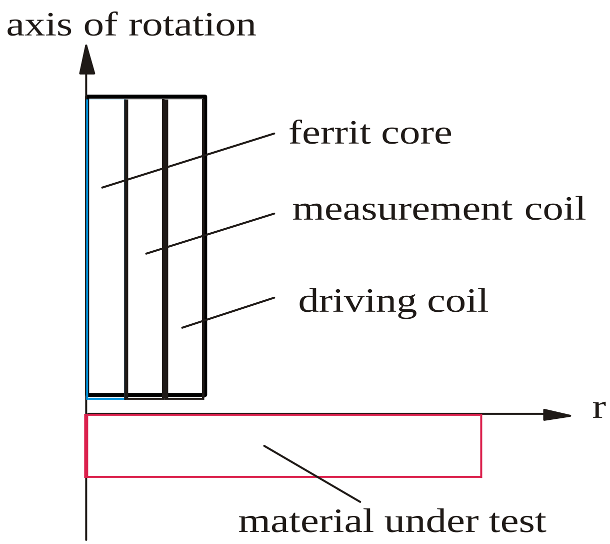
$$\mathbf{A} \times \mathbf{n} = 0$$

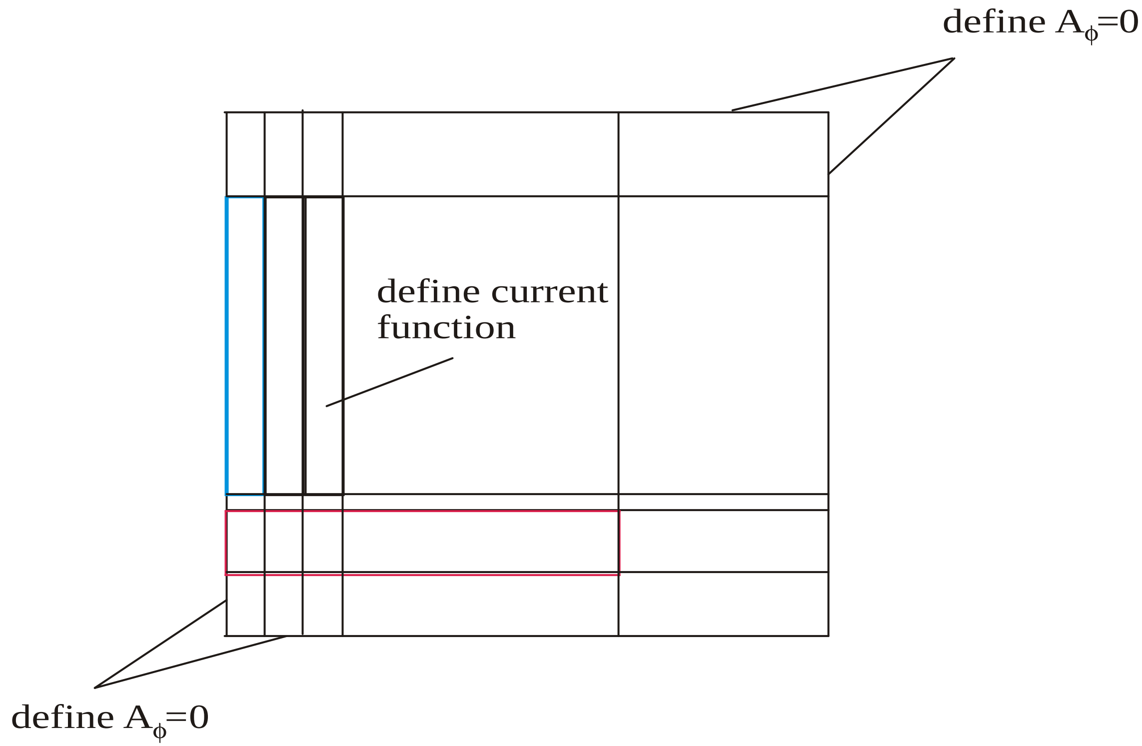
- Interface conditions:

$$[\mathbf{A} \times \mathbf{n}] = 0$$
$$[\nu \, \mathbf{n} \times \nabla \times \mathbf{A}] = 0$$

Eddy Current Sensor (I)

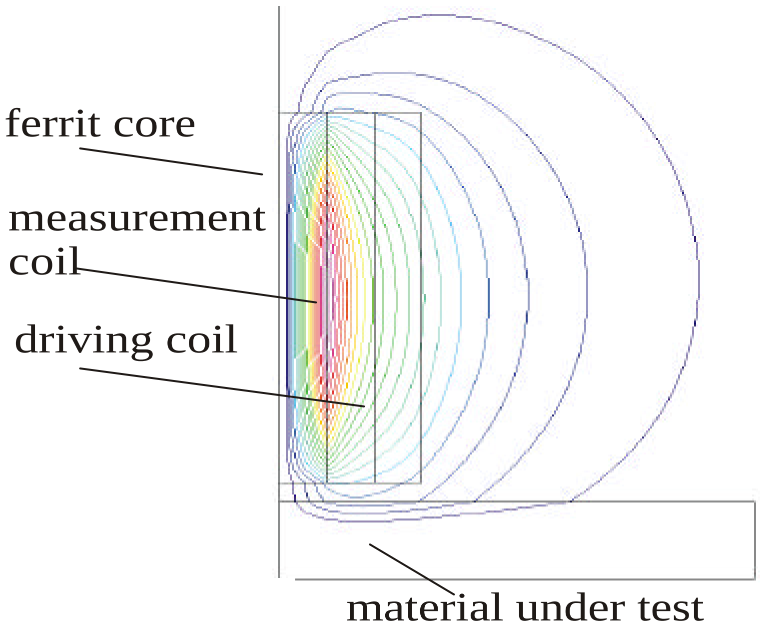
- Principle setup
- Finite Element model



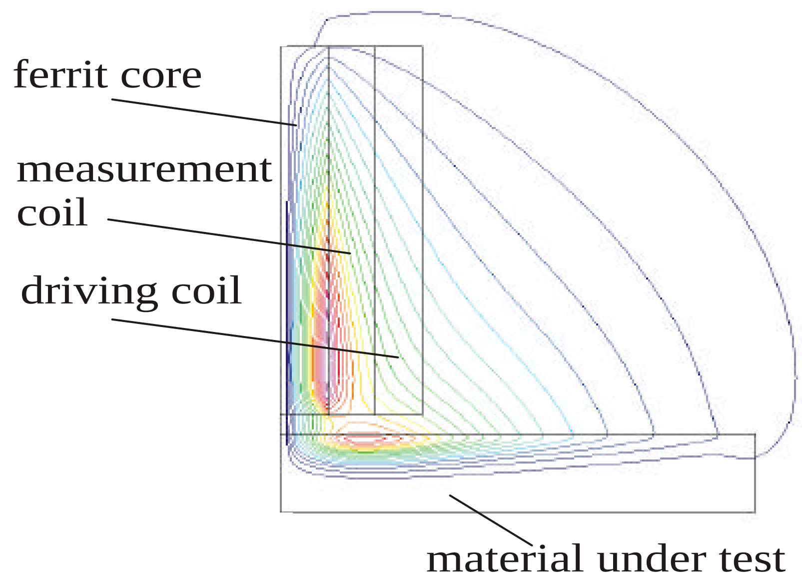


Eddy Current Sensor (II)

- Magnetic field (driving current is maximal)



- Magnetic field (driving current is zero)



Eddy Current Sensor (III)

- Penetration depth

$$\delta = \frac{1}{\sqrt{\pi f \gamma \mu}}$$

f	frequency of driving current
γ	conductivity of test material
μ	permeability of test material

element type	number of elements per penetration depth	induced voltage
linear	2	173.6 mV
linear	4	193.8 mV
linear	8	195.3 mV
quadratic	8	200.4 mV

Magnetic Force (I)

- Magnetic Energy

$$W_{\text{mag}} = \frac{1}{2} \int_{\Omega} \vec{H} \cdot \vec{B} \, d\Omega$$

$$\vec{B} = [\mu] \vec{H}$$

$\vec{H}$ magnetic field
$\vec{B}$ magnetic induction
$[\mu]$ permeability tensor

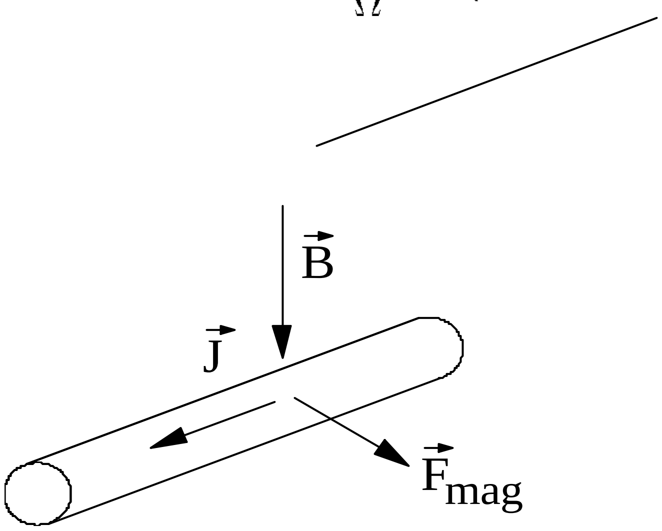
- Method of virtual displacement

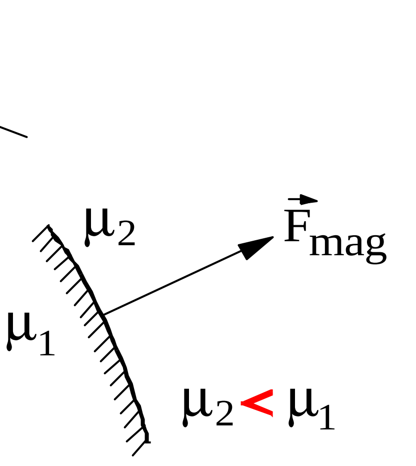
$$dW_{\text{mag}} = \vec{F}_{\text{mag}} \cdot d\vec{r}$$

Magnetic Force (II)

- Magnetic Force

$$\vec{F}_{\text{mag}} = \int_{\Omega} \left(\vec{J} \times \vec{B} - \frac{\vec{H} \cdot \vec{H}}{2} \nabla[\mu] \right) d\Omega$$





Magnetic Force (III)

- Magnetic Force Tensor

$$\mathbf{T}_{\mathrm{mag}} = \begin{bmatrix} \mu H_x^2 - \frac{1}{2}\mu|\vec{H}|^2 & \mu H_x H_y & \mu H_x H_z \\ \mu H_y H_x & \mu H_y^2 - \frac{1}{2}\mu|\vec{H}|^2 & \mu H_y E_z \\ \mu H_z H_x & \mu H_z H_y & \mu H_z^2 - \frac{1}{2}\mu|\vec{H}|^2 \end{bmatrix}$$

- Magnetic Force

$$\vec{F}_{\mathrm{mag}} = \int\limits_A \mathbf{T}_{\mathrm{mag}}\,\vec{n}\;dA$$

Electromotive Force (emf)

- Electric field

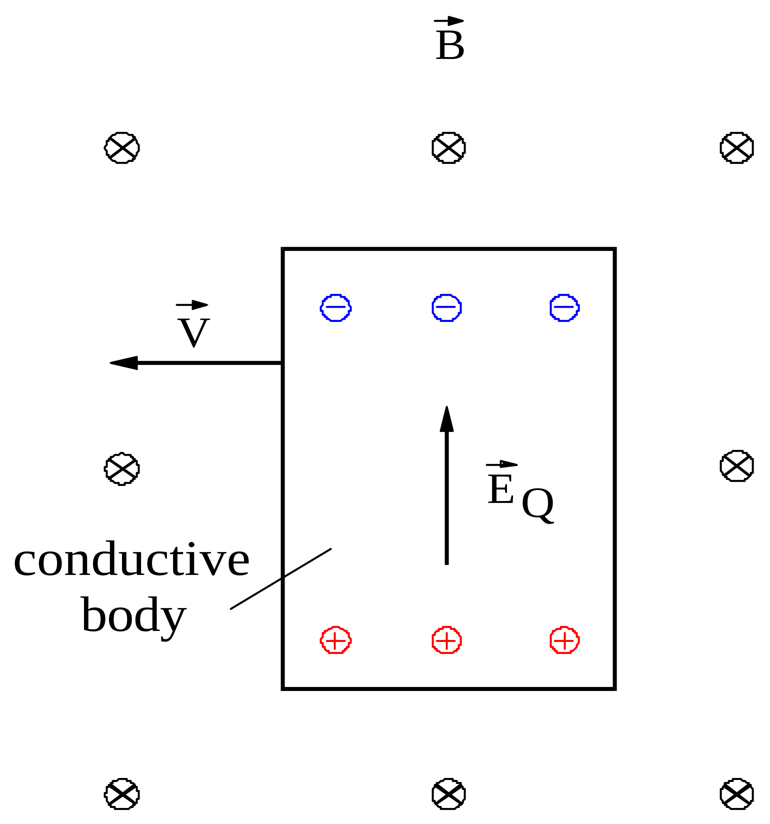
$$\vec{E}_{\mathrm{Q}} = \vec{v} \times \vec{B}$$

- Induced Voltage

$$u_{\mathrm{ind}} = \int_0^h \left(\vec{v} \times \vec{B}\right) \cdot d\vec{s}$$

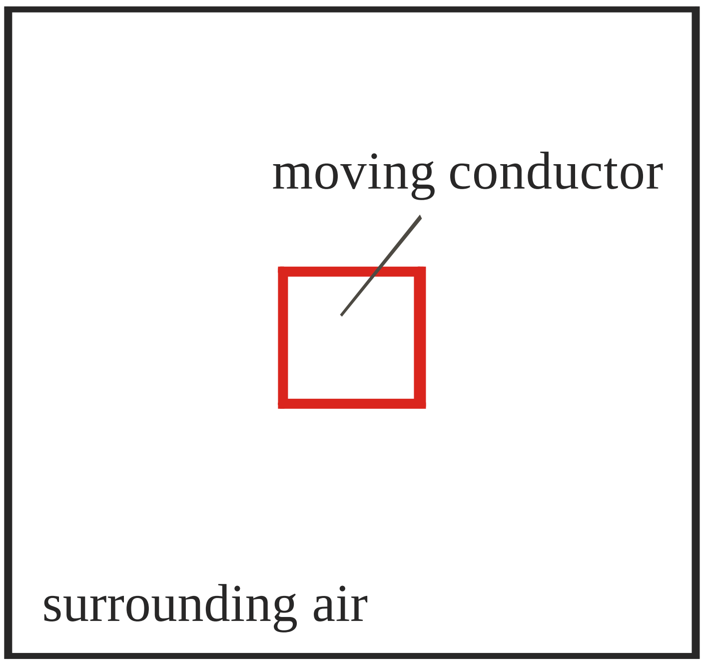
- Eddy current

$$\vec{J}_{\mathrm{w}} = \gamma \left(\vec{v} \times \vec{B}\right)$$

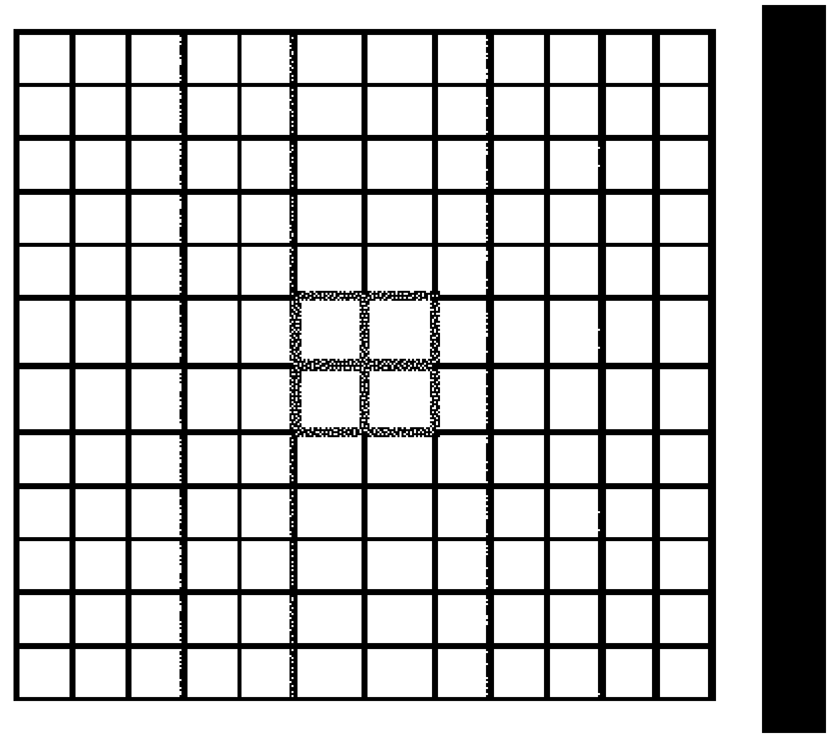


Moving Body in a Magnetic Field (I)

- Moving conductor in a magnetic field

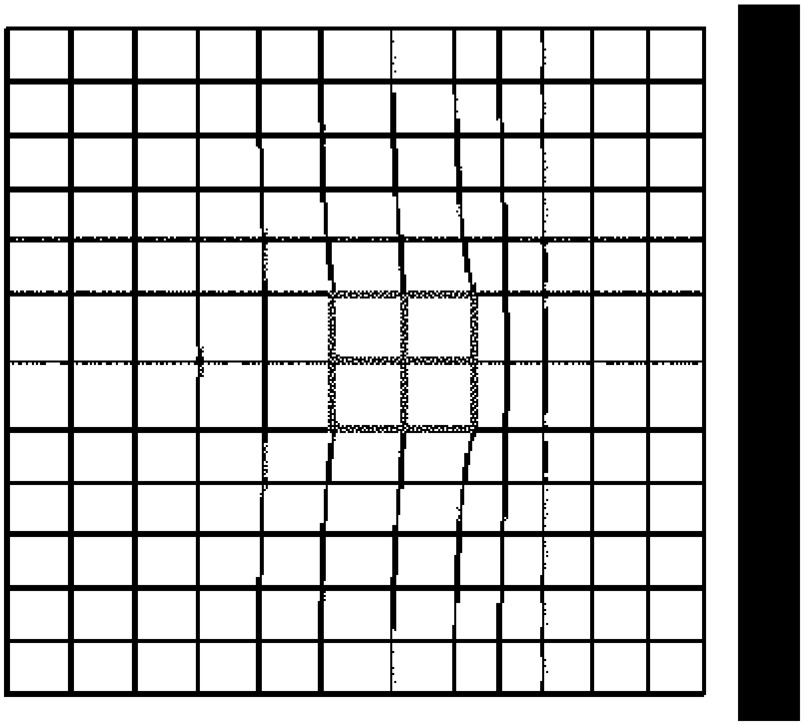


- Finite-Element-Model

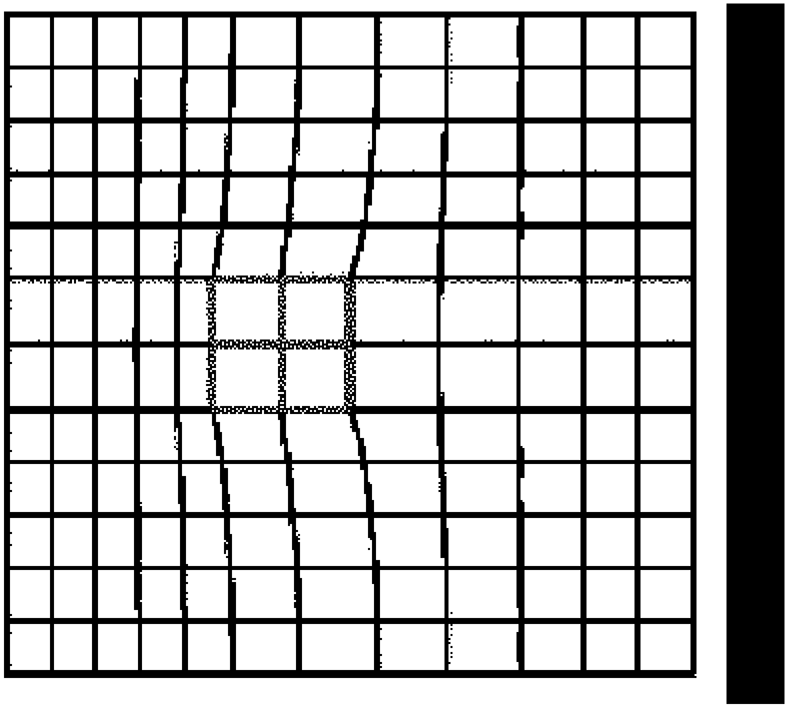


Moving Body in a Magnetic Field (II)

- Moving mesh technique



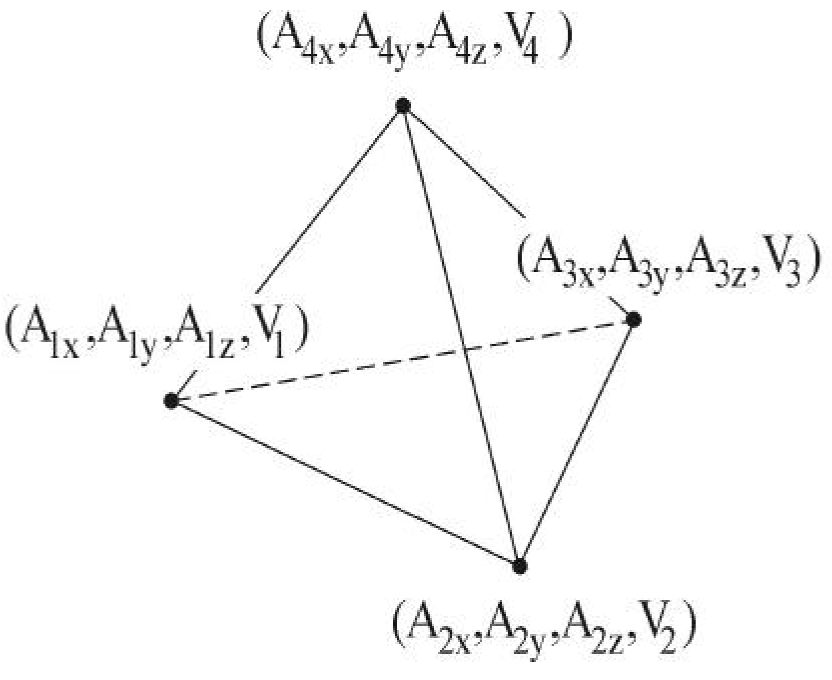
Position 1



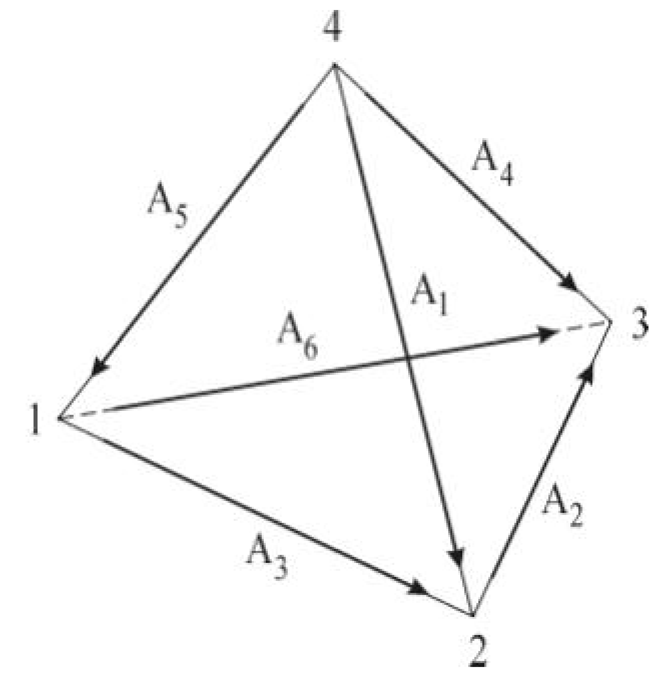
Position 2

Finite Element Discretization (I)

Nodal Finite Element



Edge Finite Element



Finite Element Discretization (II)

- Finite element equation (partial time derivative)

$$\mathbf{L}\{\dot{A}\} + \mathbf{P}\{A\} + \mathbf{P}_{\mathrm{v}}(\dot{u})\{A\} \;=\; \{Q\}$$

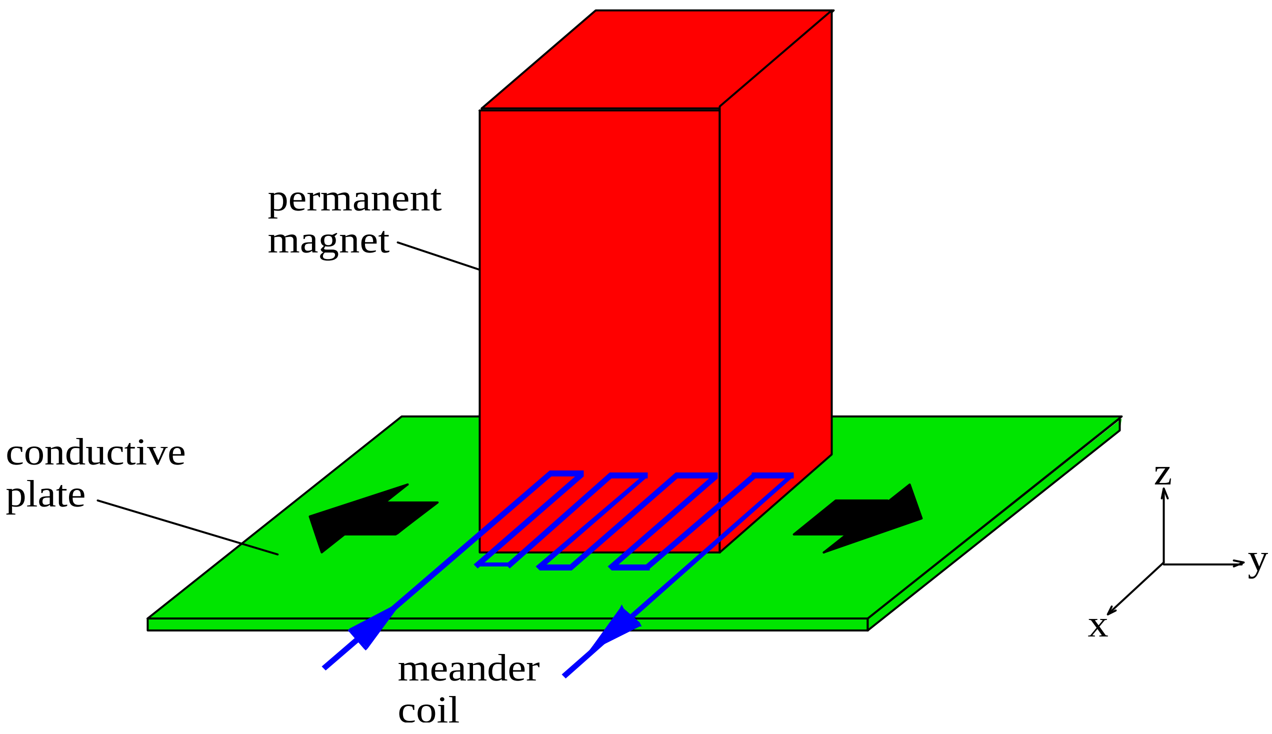
- Finite element equation (total time derivative)

$$\mathbf{L}(u)\{\dot{A}\} + \mathbf{P}(u)\{A\} + \;=\; \{Q\}$$

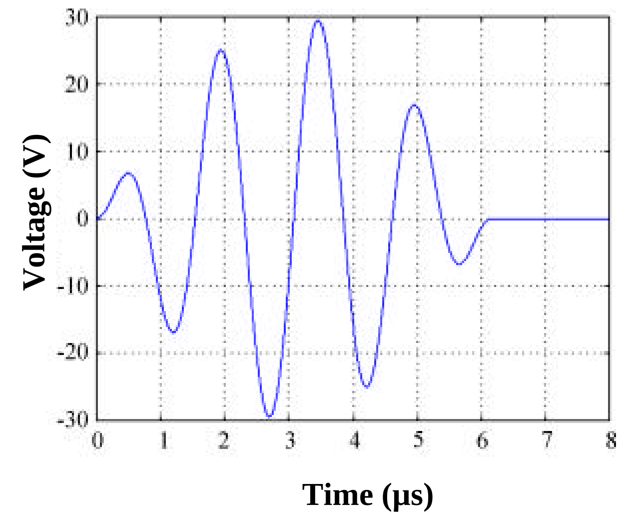
$\mathbf{L}$	conductivity matrix	$\{Q\}$	nodal source vector
$\mathbf{P}$	standard permeability matrix	$\{A\}$	nodal vector of magnetic
$\mathbf{P}_{\mathrm{v}}$	coupling permeabilty matrix		vector potential

Electromagnetic Acoustic Transducer

- Principle



- Excitation Signal



FE – Discretization

- Standard FE-discretization:

Magnetic mesh **=** Mechanical mesh

➡ 800.000 magnetic unknowns

➡ 2.400.000 mechanical unknowns

- Adapted FE-discretization:

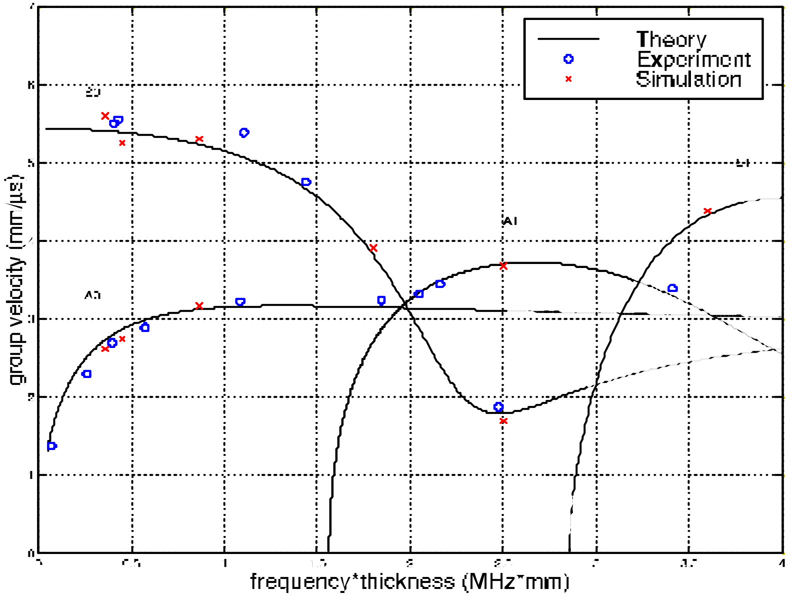
Magnetic mesh ≠ Mechanical mesh

➡ 800.000 magnetic unknowns

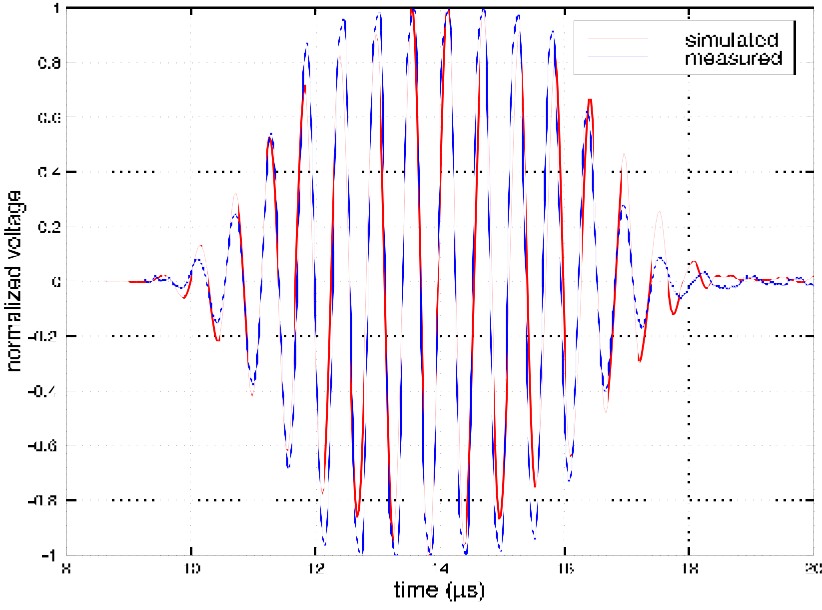
➡ 360.000 mechanical unknowns

Electromagnetic Acoustic Transducer

- Group velocity diagram



- Received voltage



Measurement of Directivity Pattern

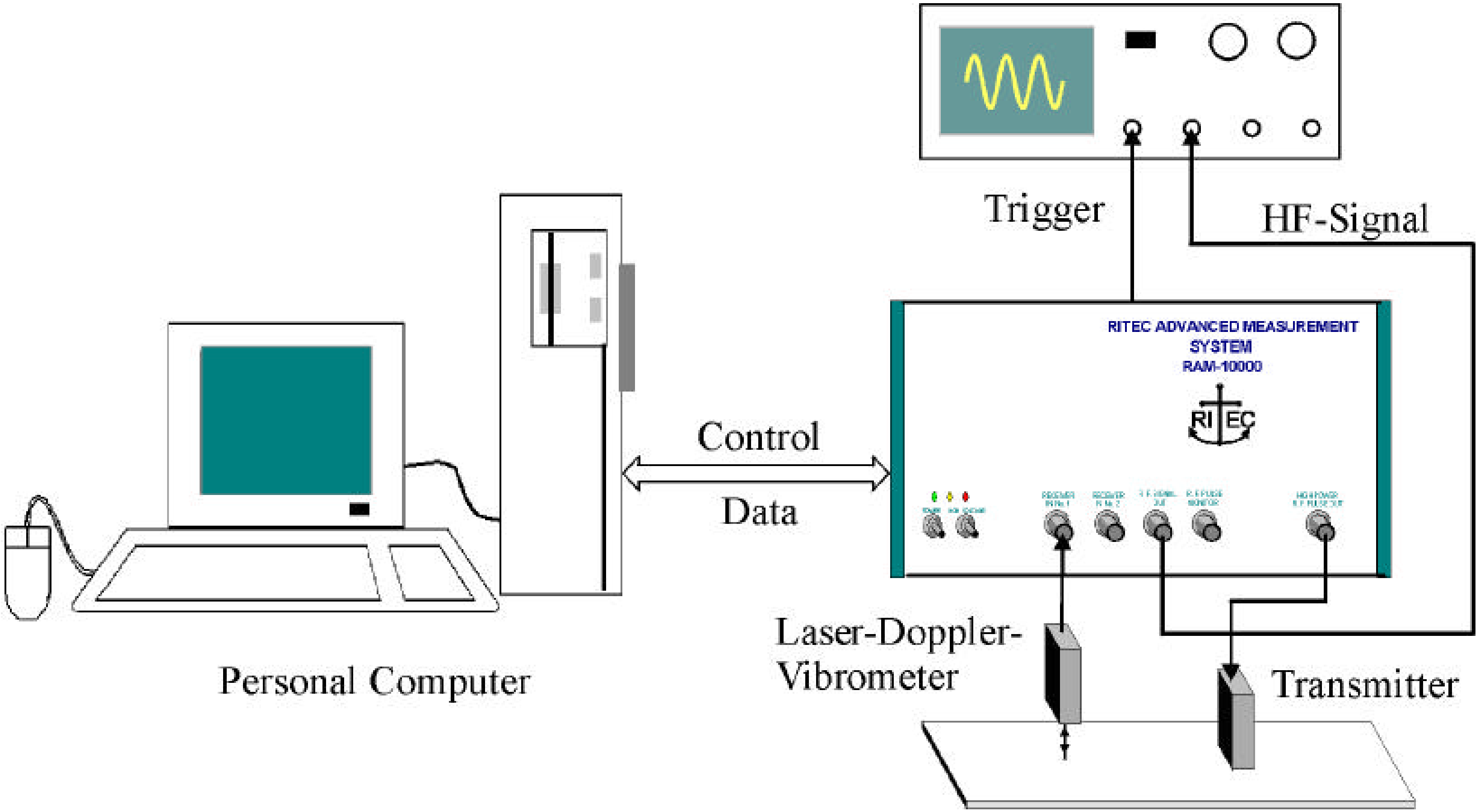
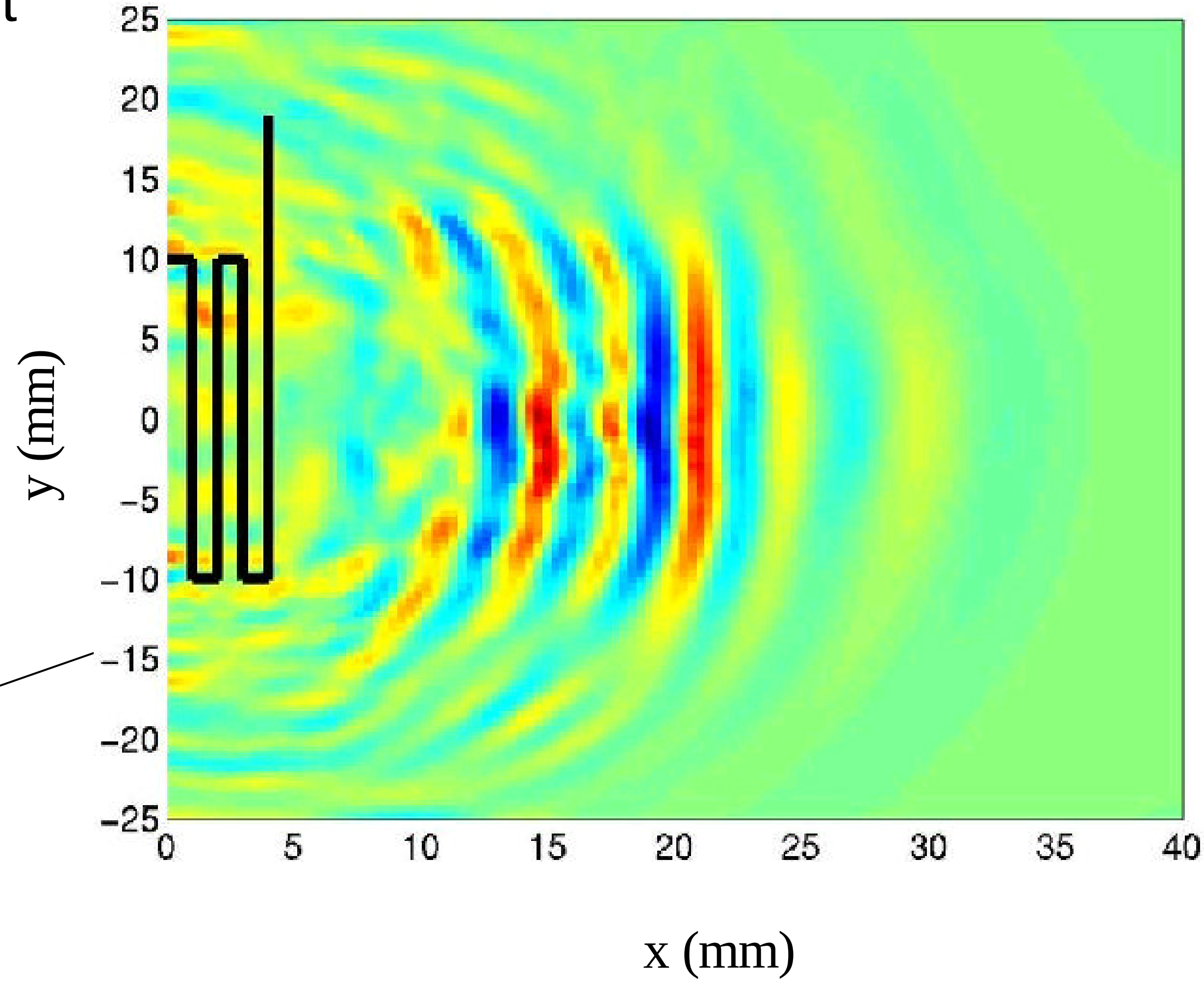


Plate Wave Propagation

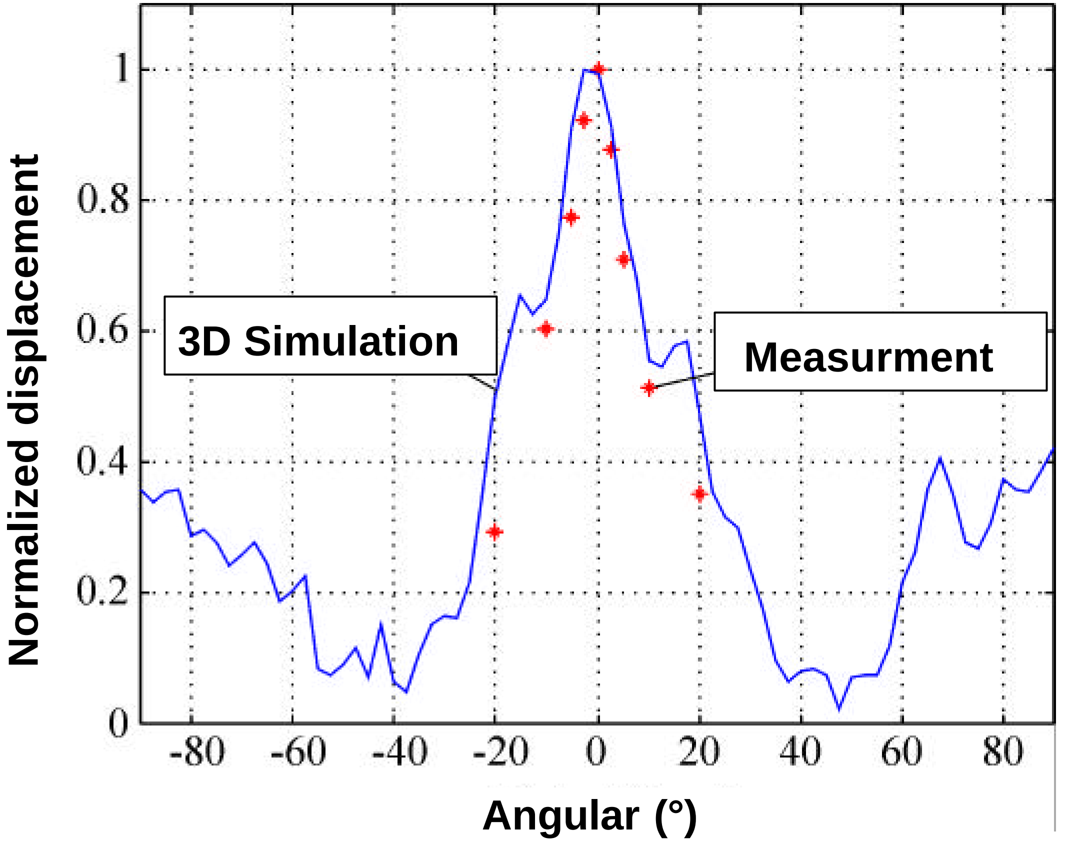
z-displacement at surface



Symmetry-plane

Measured and Simulated Results

- Directivity pattern for f=0,65 MHz:



Comparision of Solution Time

• Magnetic system of equations (n = 800.000):

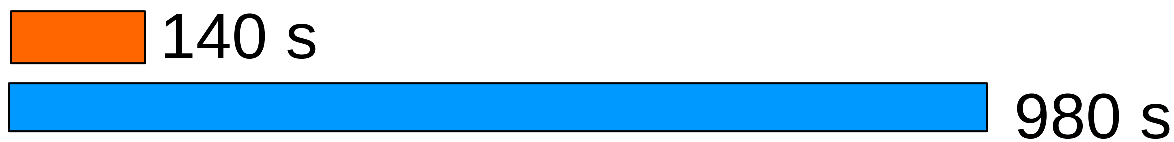
Multigrid-Solver:

ICCG-Solver:

• Mechanical system of equations (n = 360.000):

Multigrid Solver:

ICCG-Solver:



• Total elapsed CPU-time (150 time steps):

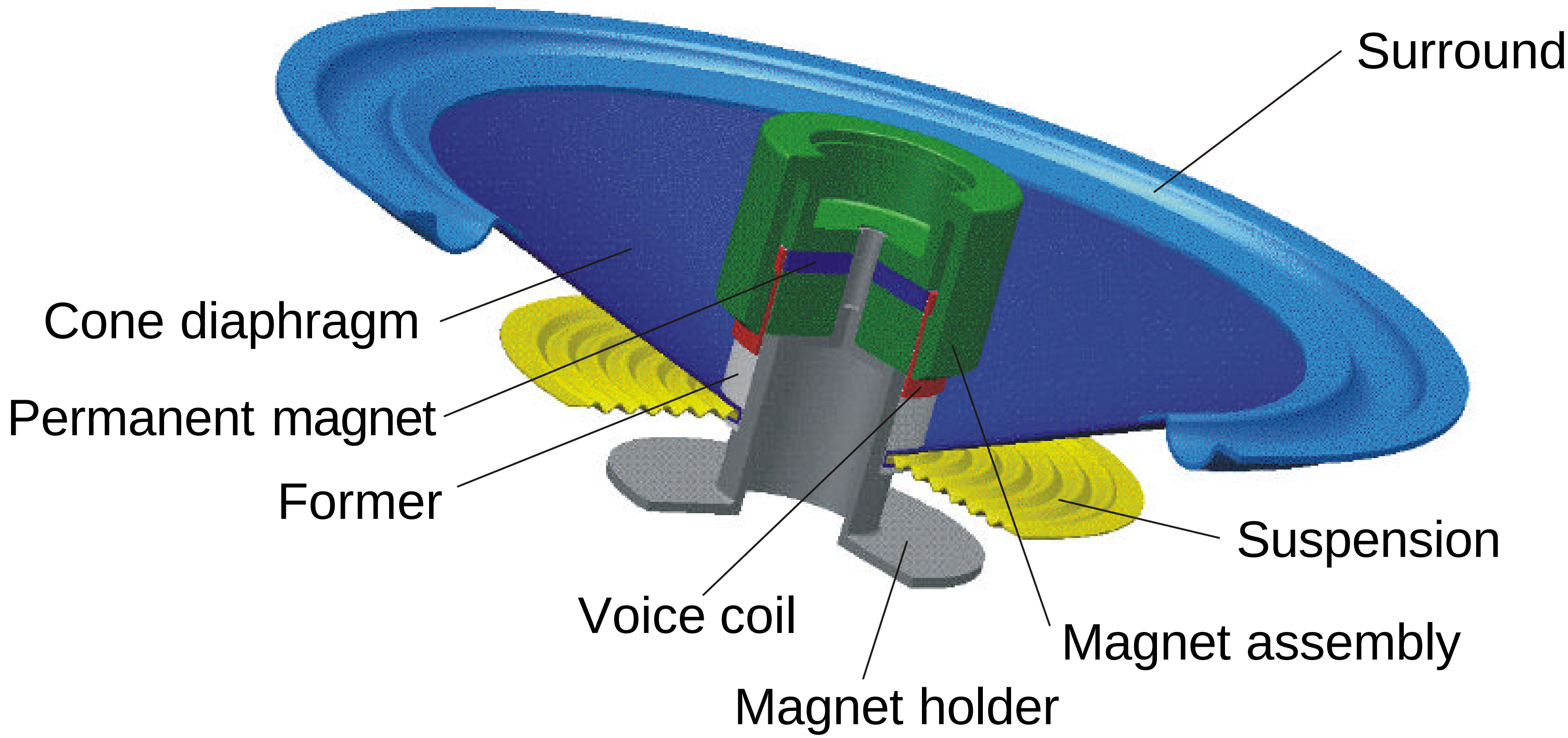
Multigrid Solver:

ICCG-Solver:

ICCG: Incomplete Cholesky Conjugate Gradient

Workstation: SGI Octane, 300 MHz

Electrodynamic Loudspeaker



Electrodynamic Loudspeaker

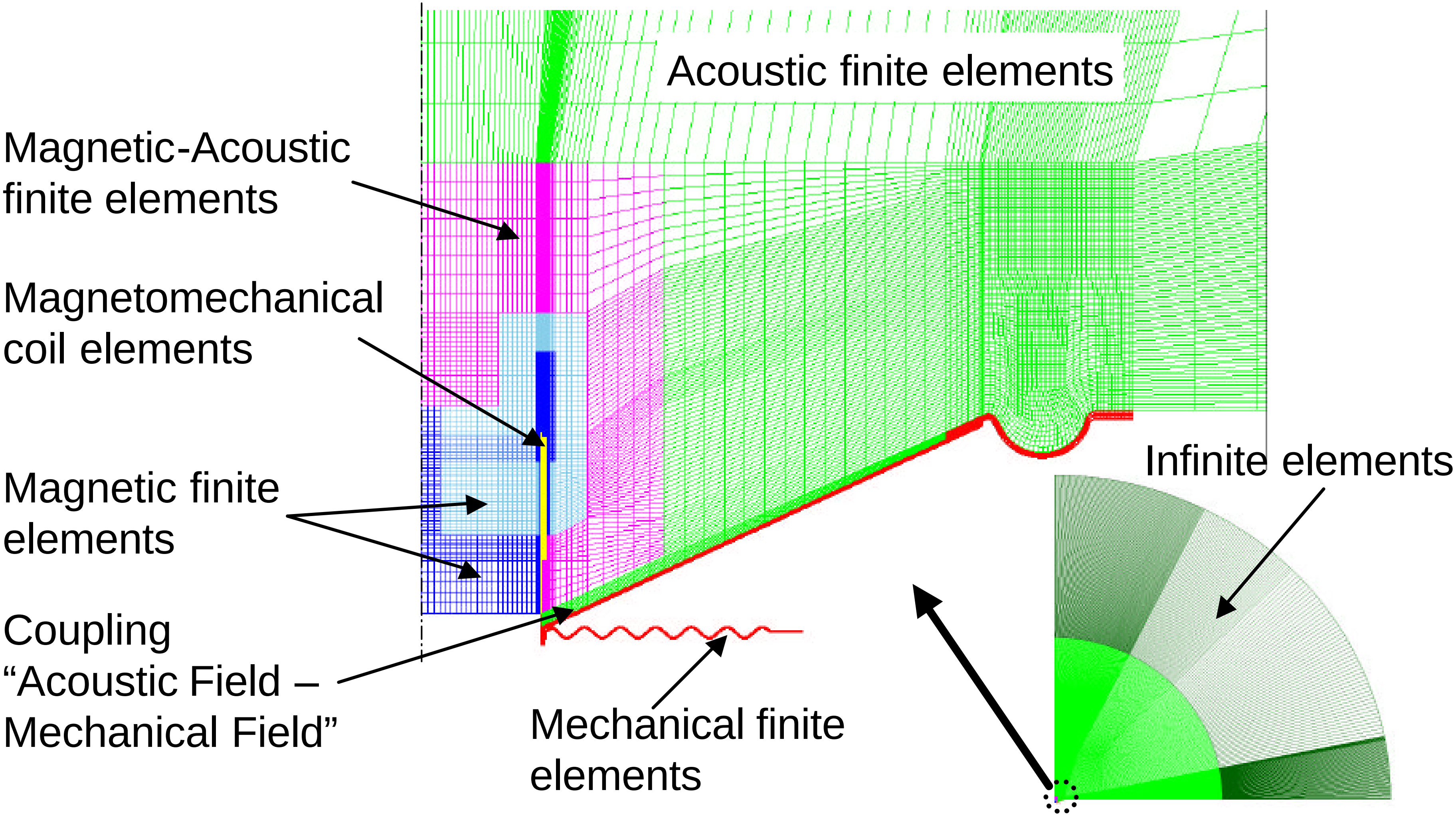
- **Design parameters**
  - Frequency dependence of axial pressure response at 1m
  - Frequency dependence of electric input impedance
- **Requirements**
  - Frequency range: 0 – 20kHz
  - Frequency solution: 2Hz

Perform a **dynamic analysis** using a short pulse excitation signal, compute the response signal and divide the fourier transformations of output and input signal

- Compute harmonic distortion

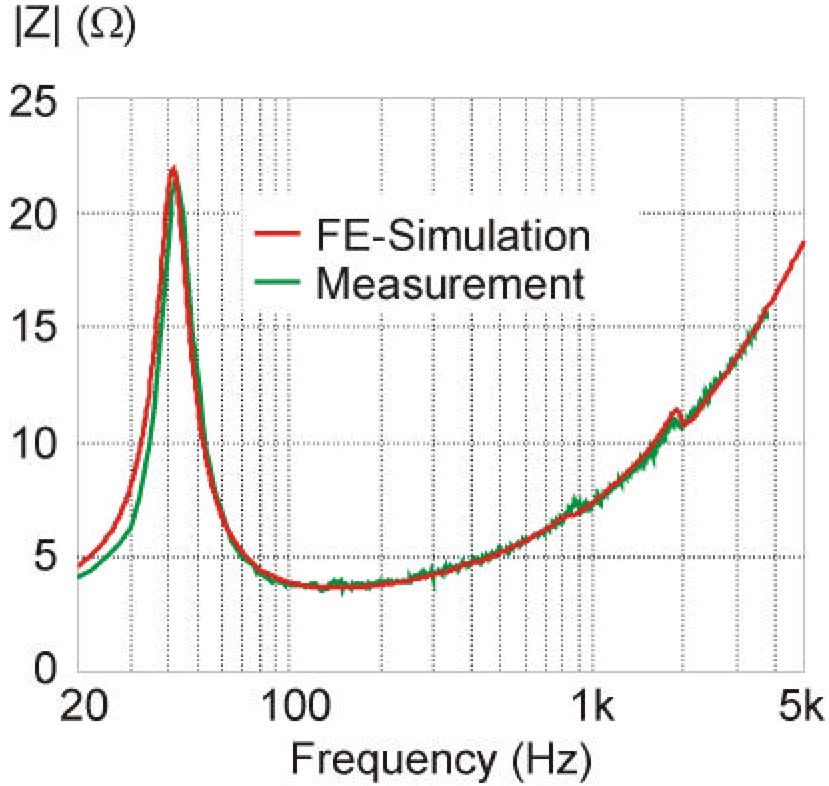
Dynamic analysis using sine- excitation

Finite-Element-Model

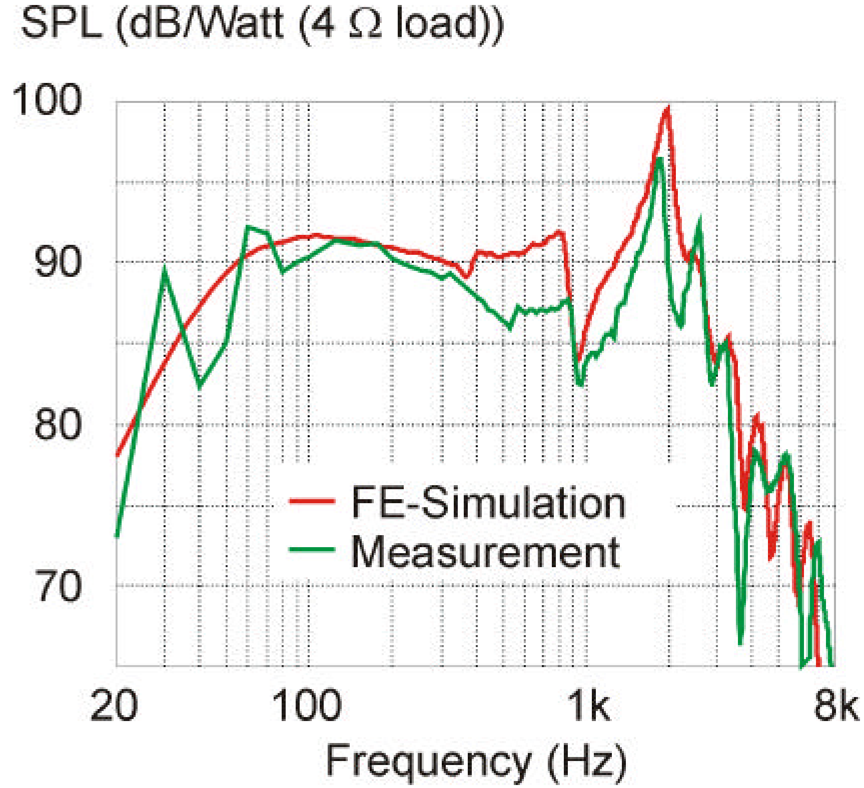


Simulated and Measured Results (I)
Small-Signal-Behavior

Electrical input impedance



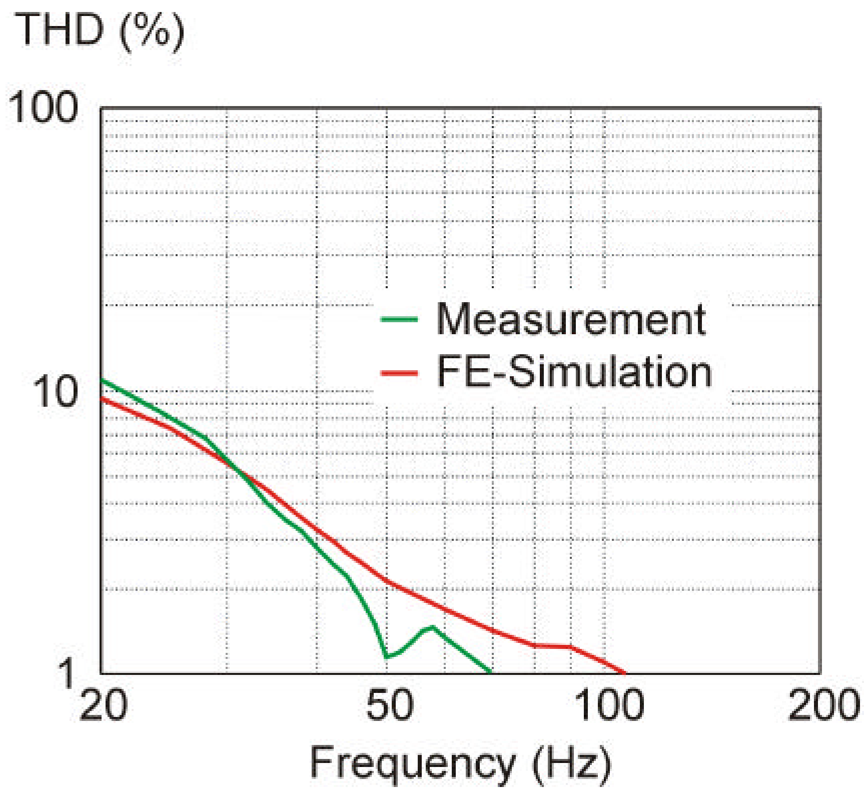
Axial pressure response at 1m
(Voltage clamping)



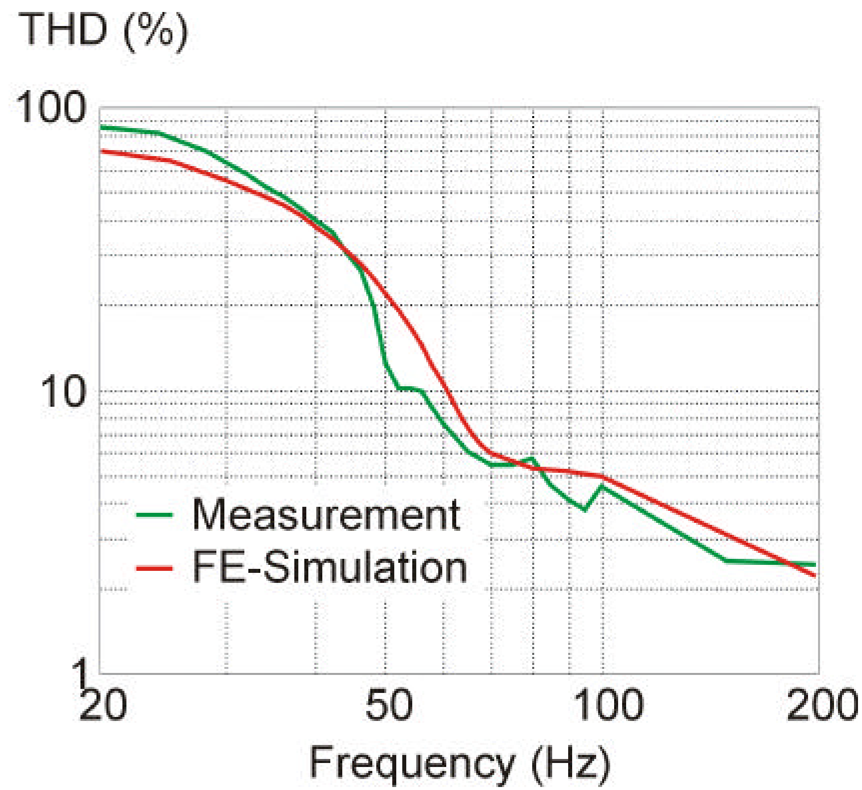
Simulated and Measured Results (II)
Large-Signal-Behavior

Total Harmonic Distortion (THD) of diaphragm acceleration

at an input power of 1 W

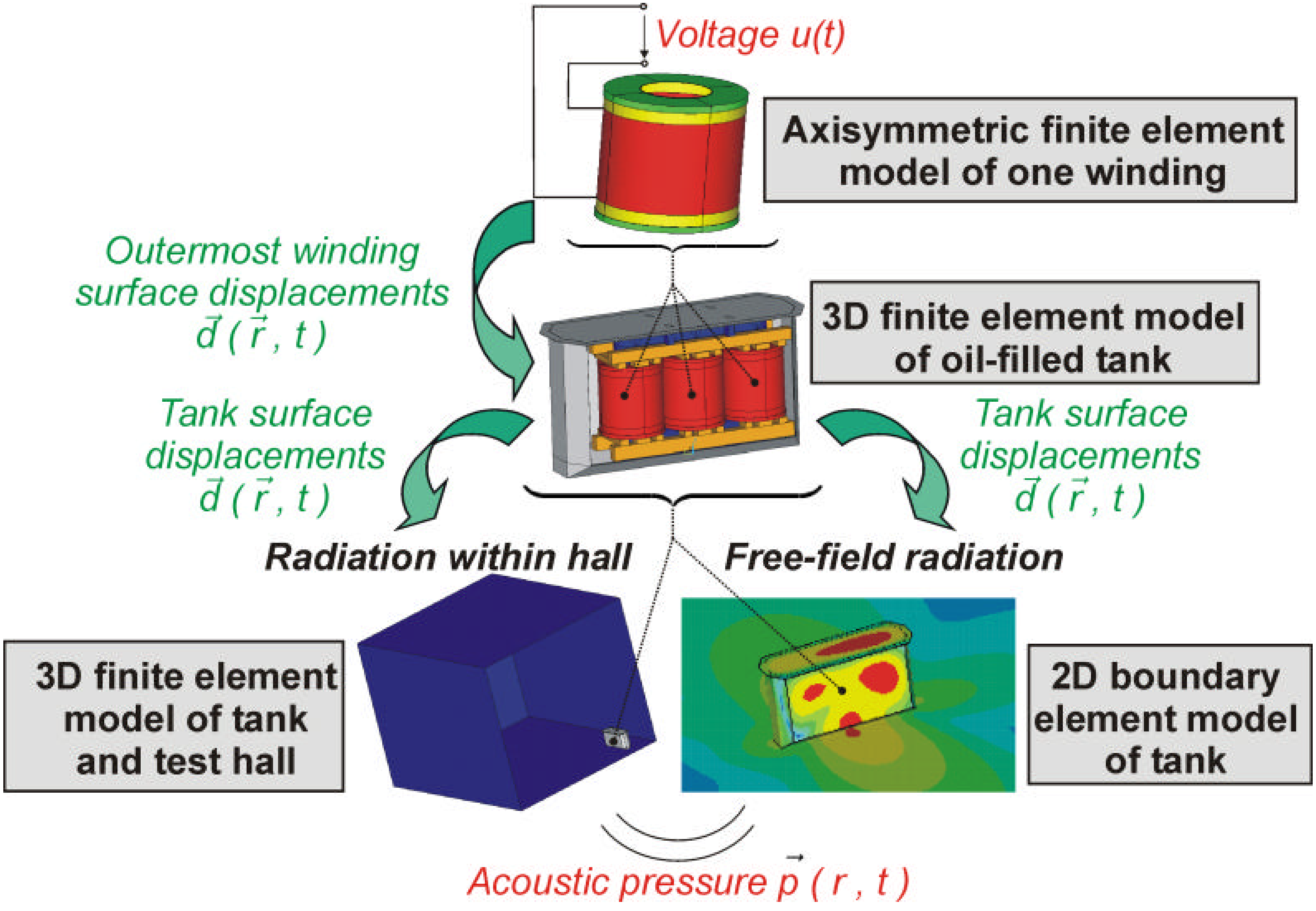


at an input power of 32 W



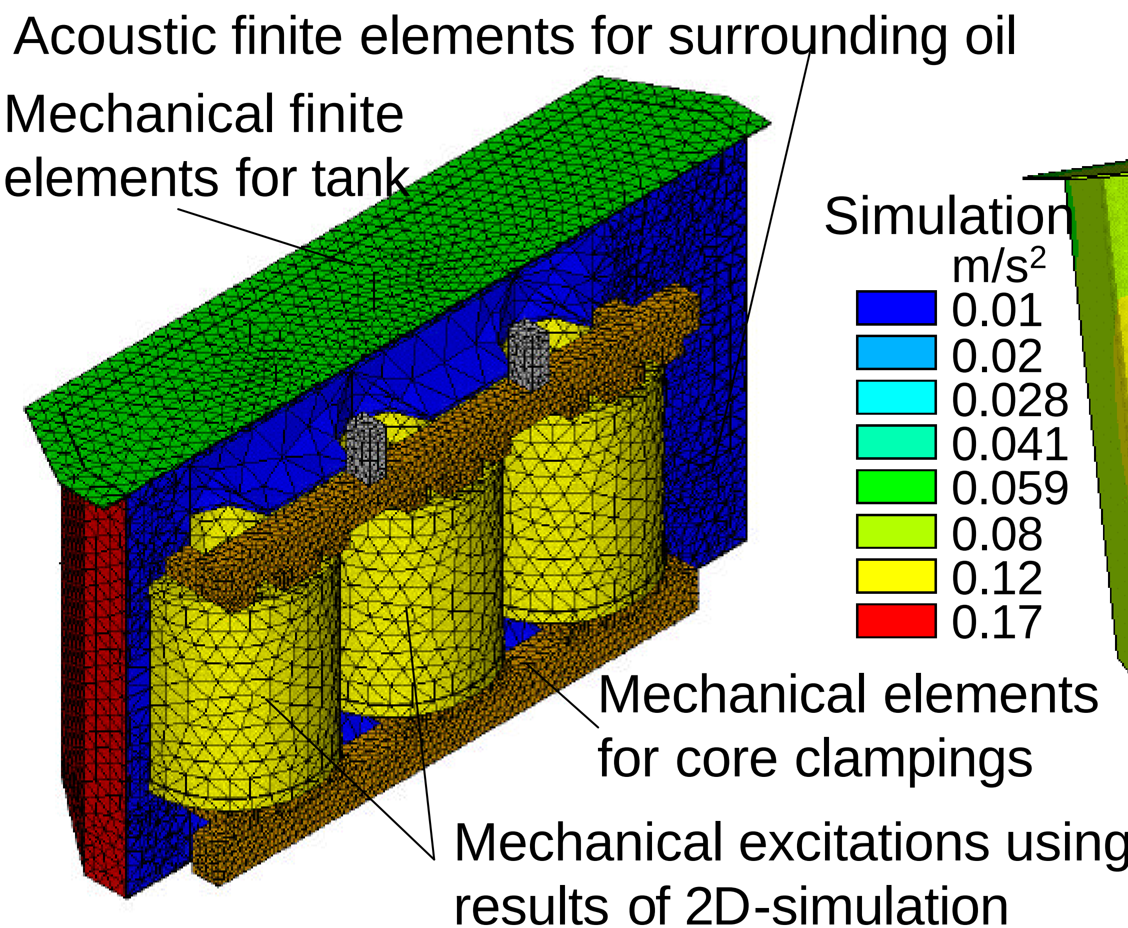
Sound Emission of Loaded Power Transformers

Overview of the modeling scheme

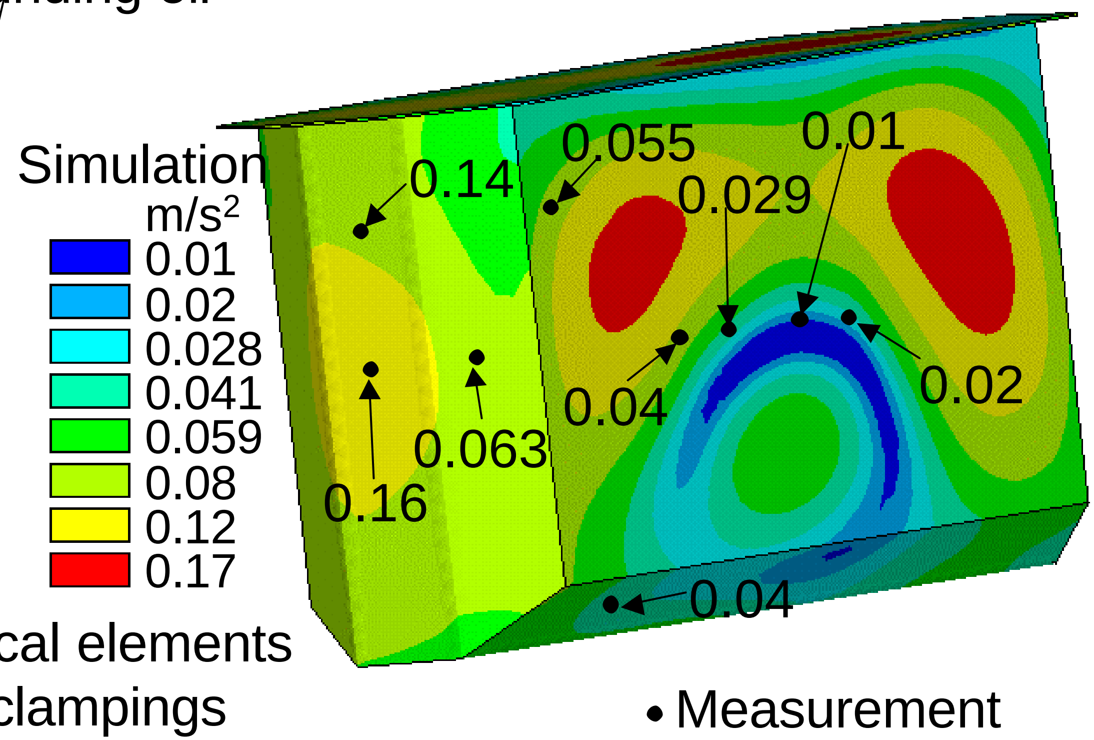


FE Modeling of Oil-filled Tank

3D finite element model



Measured and simulated tank accelerations



Simulated and Measured Sound Power Levels (SPL)

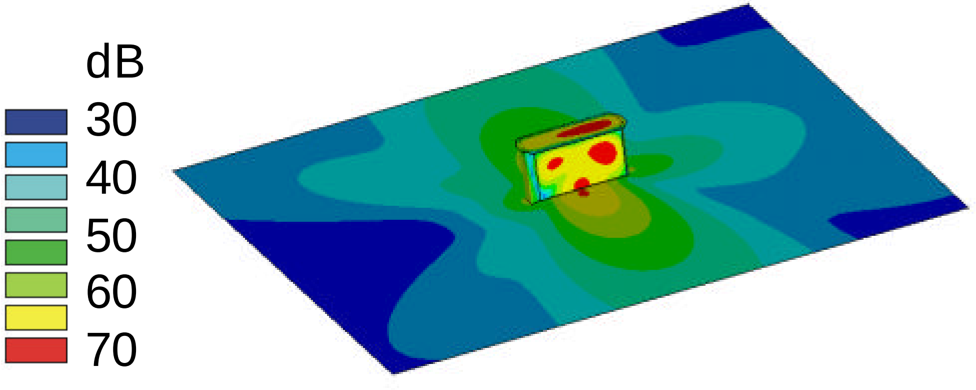
Radiation within test hall

	SPL
Sound pressure measurement	68.0 dB(A)
FEM-Simulation	66.5 dB(A)
Prediction formulas	60.5 dB(A) or 63.5 dB(A)

Free-field radiation

	SPL
Sound intensity measurement	61.0 dB(A)
BEM-Simulation	59.0 dB(A)

Simulated sound pressure levels

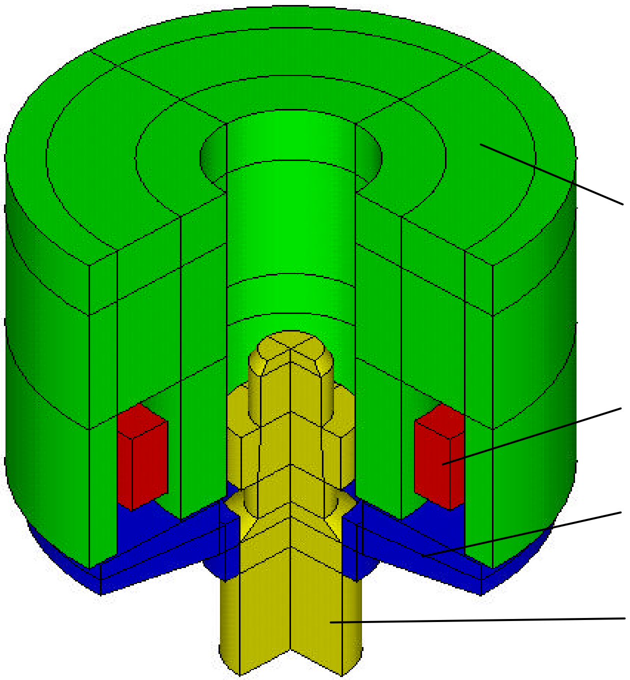


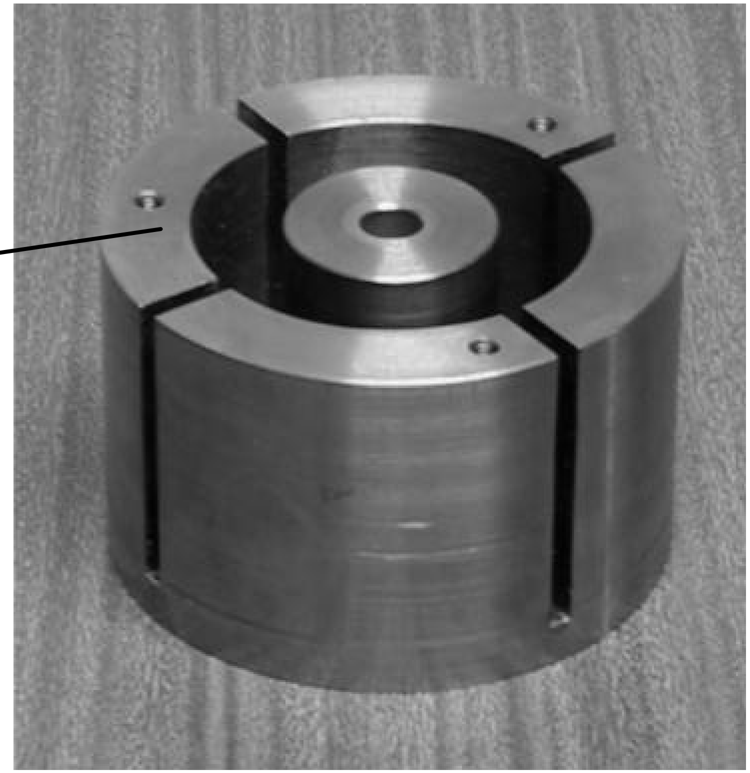
Electromagnetic Valve

- **Objective**
  - Evaluate switching time
- **Requirements**
  - Solution for mechanical field
  - Solution for magnetic field
  - Electromagnetic force calculation
  - Moving body in a magnetic field
  - Circuit coupling (coil modeling)

Electromagnetic Valve

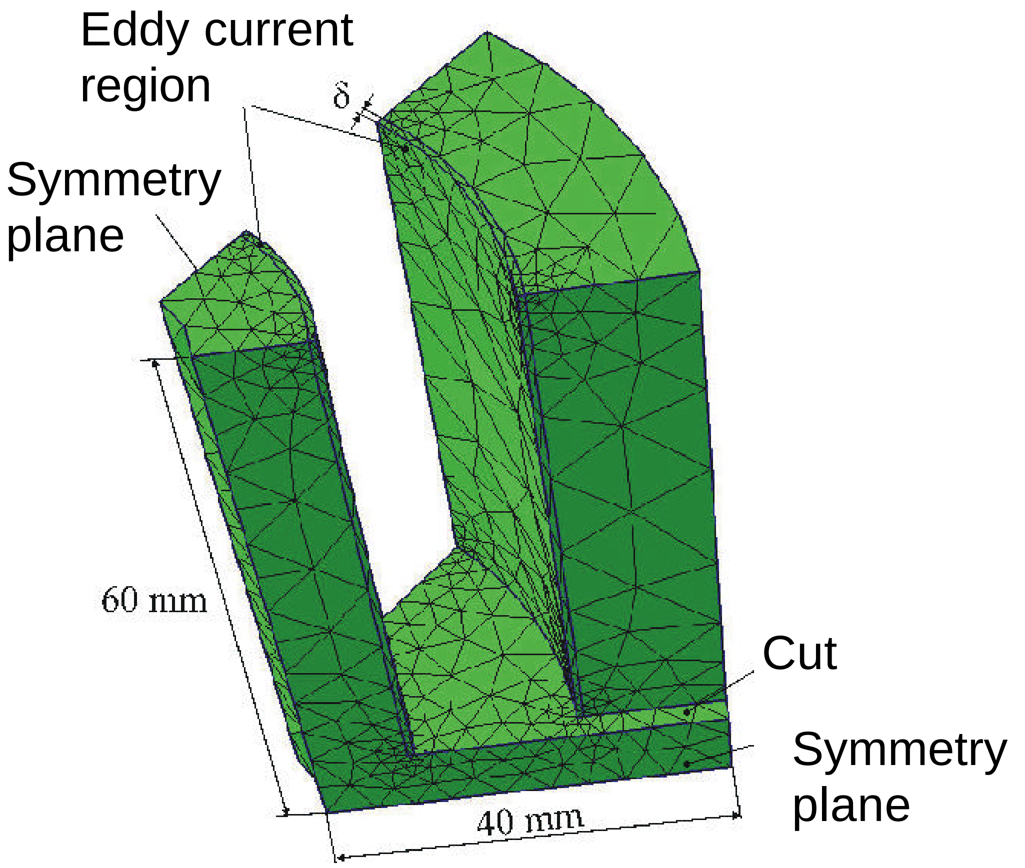
- Principle





Simulationsmodell

- FE-Model of magnetic pot



- Eddy current domains

Penetration depth:

$$\delta = \frac{1}{\sqrt{\pi\, \mu_r \mu_0\, f \gamma}} = 122.5\ \mu m$$

➡ Different geometric dimensions

➡ Magnetic mesh has to be finer

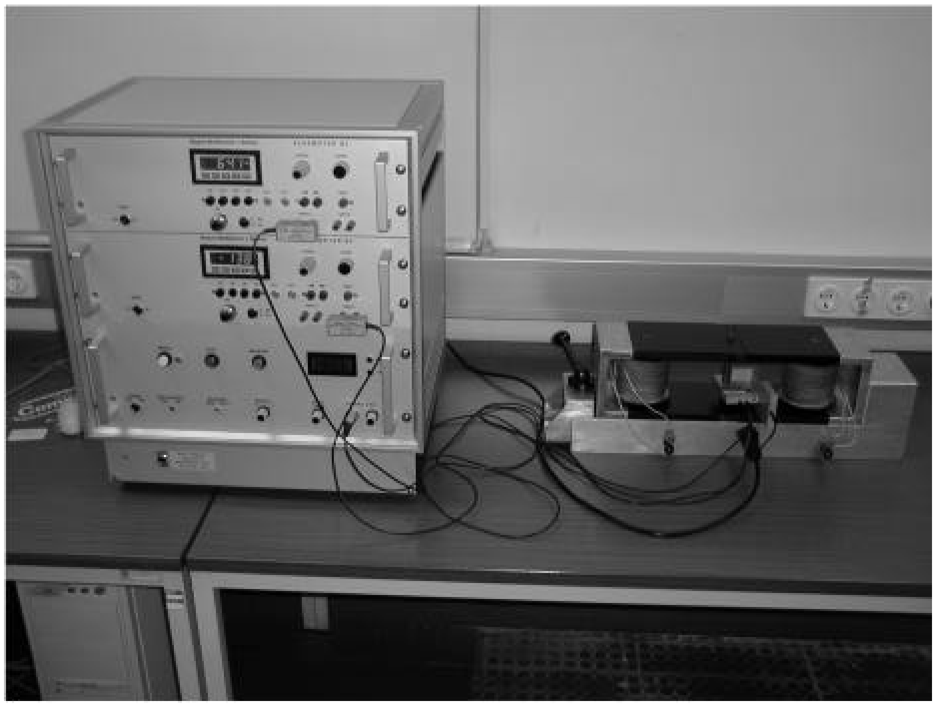
f... Frequency

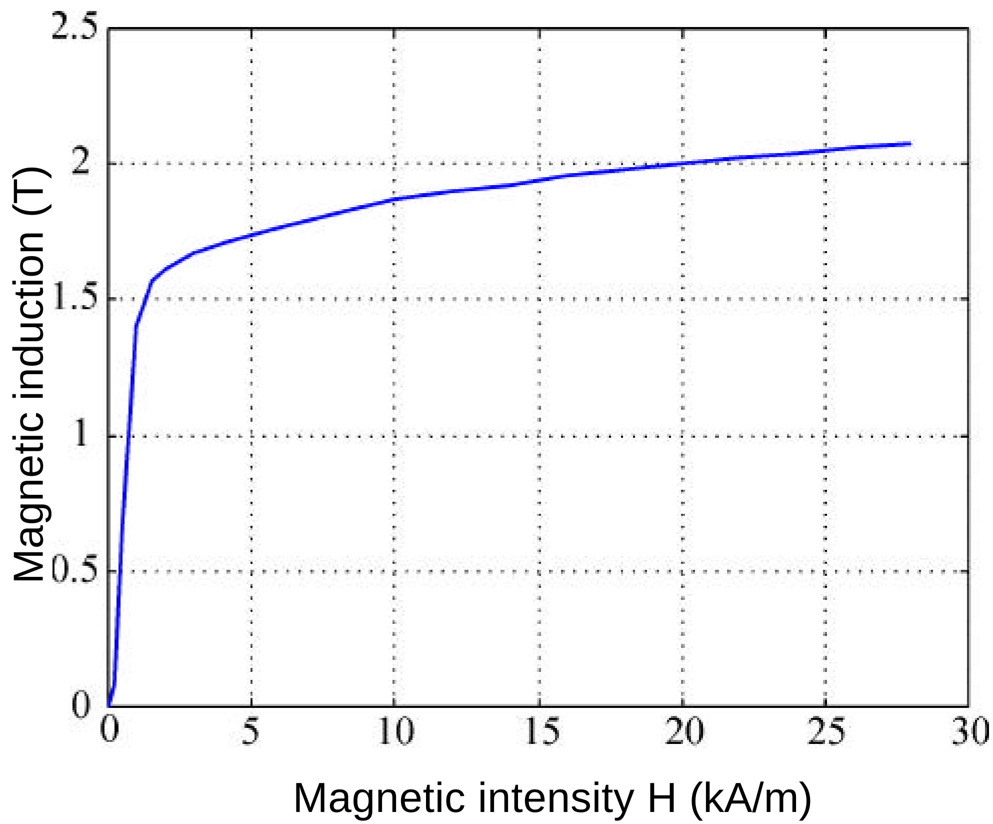
μ... Permeability

γ... Conductivity

Measurment of BH-Curve

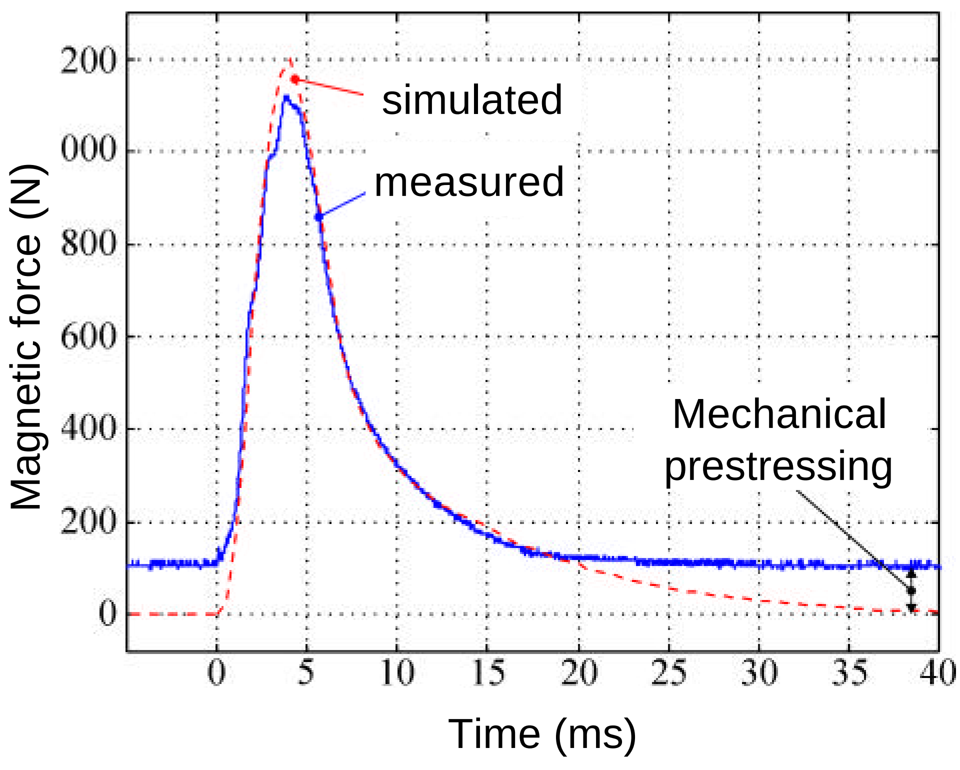
- Measurment setup
- Measured BH-curve



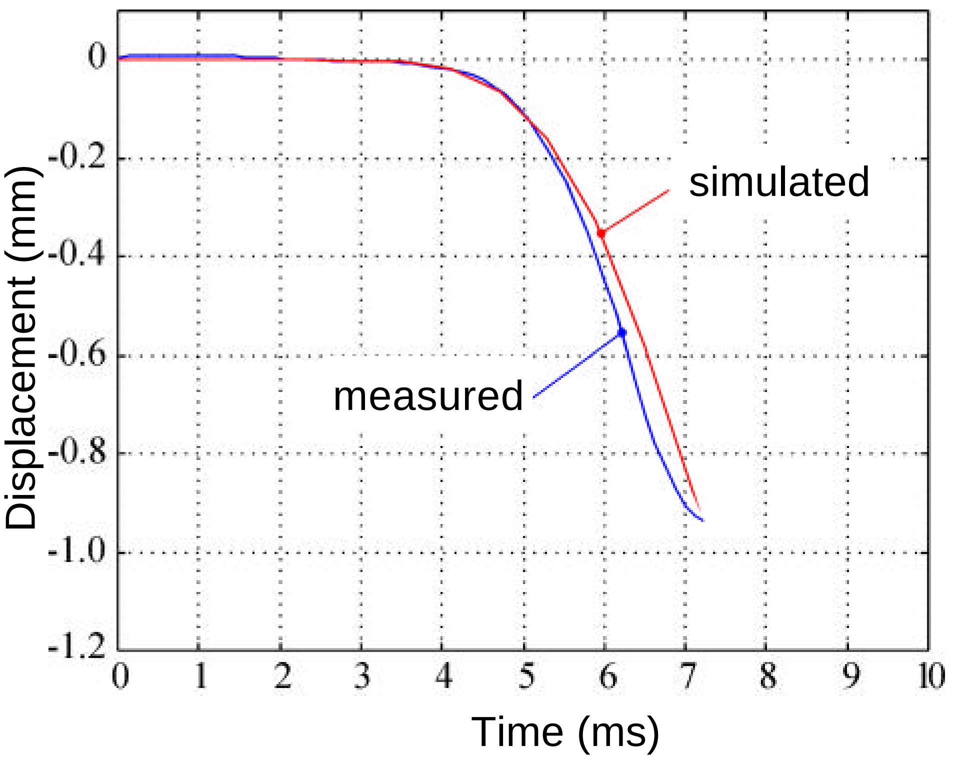


Measured and Simulated Results

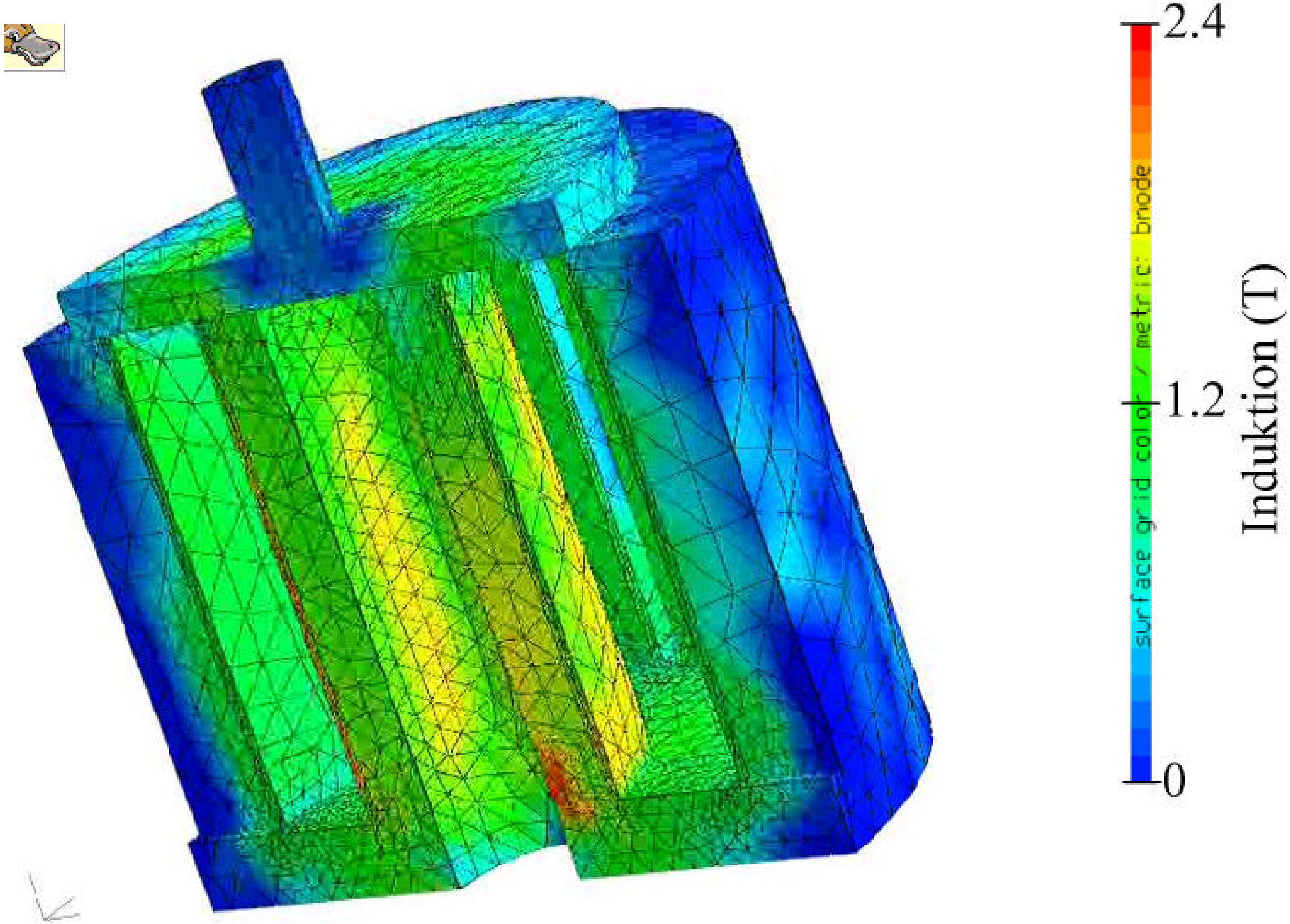
- Magnetic force (fixed tie plate)



- Moving tie plate



Magnetic Induction and Movement of Tie Plate



The End